Mathematical Journal of Modelling and Forecasting
Vol. 4, No. 1, June 2026, pp. 27-37

Optimizing the Distribution of Cow Skin Crackers at UMKM Putra-Putri Agli Using the Min-Plus Algebra Method for Shortest Route Determination

Marliana[1], Andika Ellena Saufika Hakim Maharani[2,*], and Muhammad Rijal Alfian[3]

[1,2,3] Department of Mathematics, University of Mataram, Mataram
[*] a.ellena.saufika@staff.unram.ac.id

Abstract. This study applies Min–Plus Algebra to model and analyze the distribution network of UMKM Putra-Putri Agli, a small enterprise in West Lombok engaged in the production and distribution of cowhide crackers. The objective of this research is to analyze the distribution network and identify shortest-path relationships between distribution locations based on actual distance data. The distribution system is represented as an undirected weighted graph, where nodes correspond to distribution locations and edge weights represent the distances between locations obtained from Google Maps. The analysis is conducted by constructing a distance matrix and applying Min–Plus Algebra operations to compute successive matrix powers, leading to the formation of the closure matrix A^*. The results show that the closure matrix successfully identifies the minimum distances between all pairs of distribution locations and provides the basis for deriving a distribution route with a total distance of 88.1 km. The findings also indicate that the network's structure and connectivity significantly influence route formation, as some locations can only be reached via intermediate nodes. The novelty of this study lies in the application of Min–Plus Algebra to an UMKM distribution network using actual field data and a network structure characterized by limited connectivity. The results demonstrate that Min–Plus Algebra provides a systematic algebraic framework for analyzing shortest-path relationships in small-scale distribution networks. However, the model is limited to static distance-based analysis and does not consider dynamic factors such as traffic conditions, travel time variations, or vehicle capacity constraints.

Keywords: Min–Plus Algebra, shortest path, distribution network, UMKM distribution.

1 Introduction

Distribution efficiency is one of the most important aspects of logistics and transportation systems because it directly affects operational costs, delivery time, and service quality. In Micro, Small, and Medium Enterprises (*Usaha Mikro, Kecil, dan Menengah/* UMKM), inefficient distribution practices may lead to increased transportation costs and fuel consumption. In many cases, distribution decisions are still based on experience rather than systematic optimization, which can result in inefficient routing patterns [1].

Research on shortest path optimization and distribution route planning has been widely conducted in transportation and logistics systems. Previous studies have applied various approaches, such as saving matrix methods [1], shortest path algorithms in warehouse systems [2], computational logistics models [3], and heuristic optimization techniques [4], [5]. Further developments also extend to more complex network conditions, including multimodal transportation systems, which utilize the shortest route method using Excel Solver to optimize delivery routes [6]. Niu et al. also emphasized that the mathematical optimization approach has an important role in improving the efficiency of the logistics network [7]. Further research developments show that route optimization is not only applied to static networks, but also to dynamic networks. and time-dependent routing problems [8], [9]. These studies confirm that route optimization plays a crucial role in improving distribution efficiency.

From a theoretical perspective, shortest path problems have been widely studied in transportation networks, including time-dependent routing scenarios and vehicle routing problems. From a theoretical perspective, shortest path problems can be modeled using algebraic approaches such as Min–Plus Algebra (Tropical Algebra), which provides a matrix-based framework for representing network optimization problems. This approach enables the computation of shortest paths through algebraic operations on weighted graphs and has been explored in various mathematical and network optimization studies [10]. The Min-Plus algebra is suitable for modeling transport and distribution networks because it is able to represent the shortest trajectory search through minimum operations. Min-Plus algebra uses the R set ∪ {∞} with two basic operations, namely the minimum operation in place of addition and the ordinary addition operation in place of multiplication. In the context of a distribution network, each weight on the side of the graph represents the distance between locations, whereas the Min-Plus operation is used to determine the trajectory with the minimum total weight.

For this study, Min-Plus Algebra was selected because it can systematically represent the shortest path problem in matrix operation form. This method allows the relationship between distribution locations to be studied using a weighted graph structure to determine the path with the minimum total distance mathematically. This research is not intended to compare the performance of the various shortest path algorithms but to study the application of Min-Plus Algebra in UMKM distribution networks based on actual data. In addition, Joswig and Schröter discuss the application of tropical geometry-based shortest path algorithms in network optimization [11]. In addition. Masing et al. suggest that the tropical approach can be used to determine transport routes efficiently [12], whereas Bhatia et al. explain that shortest path optimization makes an important contribution to improving distribution efficiency [13]. Further development was also carried out by Zuzic et al. through graph-based distributed shortest path [14], as well as Lee et al., which examines the application of tropical optimal transport in network systems and mathematical optimization [15].

While research on distribution route optimization and shortest-path problems has developed rapidly, most previous studies have focused on large-scale transportation networks, industrial logistics systems, or simulation models with relatively regular network structures. However, studies specifically applying Min-Plus Algebra to UMKM distribution systems using actual distance data remain limited. In addition, UMKM distribution networks often face constraints related to road accessibility and uneven connectivity between locations. Therefore, a mathematical approach capable of representing these network characteristics systematically is required.

This research aims to apply Min-Plus Algebra to analyze the distribution network of UMKM Putra-Putri Agli and identify shortest-path relationships between distribution locations based on actual network conditions. The distribution network is represented as an undirected weighted graph, with distance data obtained from Google Maps. The novelty of this study lies in the implementation of Min-Plus Algebra in an UMKM distribution network characterized by limited connectivity and a non-uniform distribution structure. Under these conditions, some locations can only be reached through specific connecting nodes, resulting in route structures that differ from networks with multiple alternative paths. Therefore, this study demonstrates the application of Min-Plus Algebra to distribution problems in UMKM based on actual field conditions and provides a mathematical framework that can support distribution planning and decision-making.

2 Research Methods

This study aims to apply Min-Plus Algebra to analyze the distribution network and support the determination of distribution routes based on shortest-path information. The method used is a quantitative method with a mathematical approach because the research focuses on processing numerical data in the form of distances between distribution locations, which are analyzed using the Min-Plus Algebra model.

The object of the research is the distribution network of cowhide cracker products in UMKM Putra-Putri Agli, which involves one production location and several distribution destination locations. The data used consists of primary data and secondary data. Primary data was obtained through interviews with UMKM owners to identify distribution locations and delivery patterns that are commonly carried out. Secondary data is obtained from Google Maps in the form of distance data between distribution locations.

Distance data between locations is obtained using Google Maps based on the distance traveled through the available road network. Measurements are made by determining the travel route from one distribution point to another, then recording the distance displayed by Google Maps in kilometers. The data is then used as a weight on each side of the graph that represents the relationships between distribution locations.

The stages of the research carried out are as follows:

1. Literature Study

This stage is carried out by examining various references related to graph theory, Min-Plus Algebra, shortest path, as well as previous research relevant to distribution route optimization.

In Min-Plus Algebra, the addition operation is expressed by the symbol ⊕ and is defined as [10]:

$$a \oplus b = min(a, b) \tag{1}$$

whereas the multiplication operation is expressed by the symbol ⊗ and is defined as[10]:

$$a \otimes b = a + b. \tag{2}$$

For the two matrices $A = (a_{jk})$ and $B = (b_{ki})$, the Min-Plus multiplication operation is defined as[10], [11]:

$$(A \otimes B)_{ji} = min_k(a_{jk} + b_{ki}) \tag{3}$$

For two matrices A and B, each element in the resulting matrix is obtained by summing the corresponding elements of both matrices, and then selecting the minimum value of all possible combinations. In the shortest path problem, this operation is used to determine the path with the minimum total weight from one point to another in the network modeled in the form of a weighted graph. Based on this definition, matrix multiplication operations in Min-Plus Algebra are performed by replacing the addition operations on conventional matrix multiplication with minimum operations, while multiplication operations are replaced by regular additions. This operation is the basis for the minimum trajectory finding process on a distribution network modeled in the form of a weighted graph.

2. Data Collection

The data used consists of primary data and secondary data. Primary data was obtained through interviews with UMKM Putra-Putri Agli owners to identify distribution locations and product delivery patterns. Secondary data is obtained from Google Maps in the form of distance data between distribution locations. Measurements are made by determining the travel route between distribution locations and recording the distance displayed by Google Maps in kilometers.

3. Graph Modeling and Weight Matrix Formation

The distribution network is modeled in the form of a weighted directionless graph, with each location represented as a node and each relationship between locations represented as an edge that has a weight of actual distance. Based on the graph, a weight matrix A was formed, which contains the distance between distribution locations. If there is no direct relationship between locations, then the matrix element is assigned an infinite value (∞) or ε in Min-Plus Algebra.

4. Application of Min-Plus Algebra and Closure Matrix Formation

Formulation of the Min-Plus mathematical model, which includes the conversion of distance data into the form of a weight matrix $A = [a_{ji}]$, and at this stage, Min-Plus Algebra operations are performed using basic operations [10]:

$$a \oplus b = \min(a, b), a \otimes b = a + b. \tag{4}$$

Next, the Min-Plus matrix rank operation is carried out to form a closure matrix A^*. The closure matrix is used to obtain the minimum distance between all node pairs on the distribution network. In general, the closure matrix is expressed as [10]:

$$A^* = I \oplus A \oplus A^2 \oplus \ldots \oplus A^{n-1} \tag{5}$$

where I is the Min-Plus identity matrix and n denotes the number of nodes on the distribution network.

5. Determination of Optimal Trajectory

Based on the closure matrix A^*, the shortest-path relationships between nodes in the distribution network were identified. A distribution route was then constructed by considering the connectivity between nodes in the graph, allowing all distribution locations to be reached while incorporating the minimum-distance information provided by the closure matrix.

6. Visualization and Analysis of Results

The calculation results are visualized in the form of a distribution network graph and a distribution route graph. The resulting route was then analyzed based on the network structure and the shortest-path information obtained from the closure matrix A^*.

This research was carried out through several systematically arranged stages to achieve the research objectives. These stages include literature review, data collection, graph modeling and weight matrix formation, application of Min-Plus Algebra and closure matrix construction, shortest-path analysis, distribution route derivation, and visualization and analysis of the results. The sequence of research stages is presented in Fig 1.

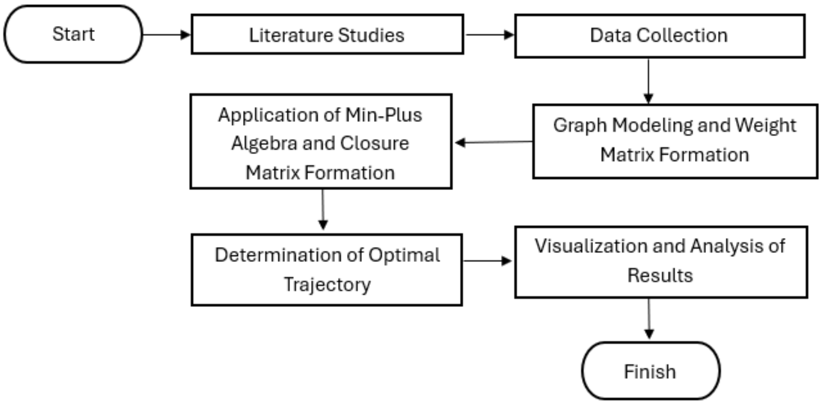

Fig 1. Research flow diagram

3 Results and Discussion

3.1 Characteristics of Distribution Networks

UMKM Putra-Putri Agli is a local enterprise engaged in the production and distribution of cow skin crackers in Tanak Tepong Village, West Lombok Regency. The distribution process is carried out from the production site to several distribution locations located in the surrounding area. Distribution vehicles cannot always travel directly between locations because of the existing road network and geographical conditions. As a result, some distribution points can only be reached through intermediate locations that serve as connecting nodes within the network.

These conditions create a distribution network with limited connectivity between certain locations. Not all distribution points are directly connected, and some routes require vehicles to pass through specific nodes before reaching their destinations. Consequently, the structure of the distribution network plays an important role in determining the routes that can be formed during the distribution process.

The characteristics of the distribution network are an important consideration in graph modeling because they influence the relationships between nodes and the possible paths within the network. Since direct connections do not exist between all locations, a mathematical representation is required to describe the network structure systematically. In this study, the distribution network is modeled as a weighted graph and analyzed using Min-Plus Algebra to determine the minimum distances between distribution locations.

3.2 Data Collection and Distribution Network Modeling

Once the characteristics of the distribution network are identified, the next step is to collect the data necessary to form a distribution network model. The data used is in the form of distribution locations and distances between locations obtained through Google Maps. Measurements are made based on the distance traveled through the available road network so that they can represent the actual distribution conditions.

The location and distance data obtained are then represented in the form of weighted directionless graphs. Each distribution location is expressed as a node, while the relationship between locations is expressed as an edge that has a weight in the form of the distance between locations. Graph modeling is used to describe the interconnectedness structure between distribution points so that it can be analyzed using Min-Plus Algebra.

On the distribution location map, the distribution destination points are marked with red markers to make it easier to identify each distribution location used in the study.

The map of distribution locations and network shapes used in the study is shown in Fig 2.

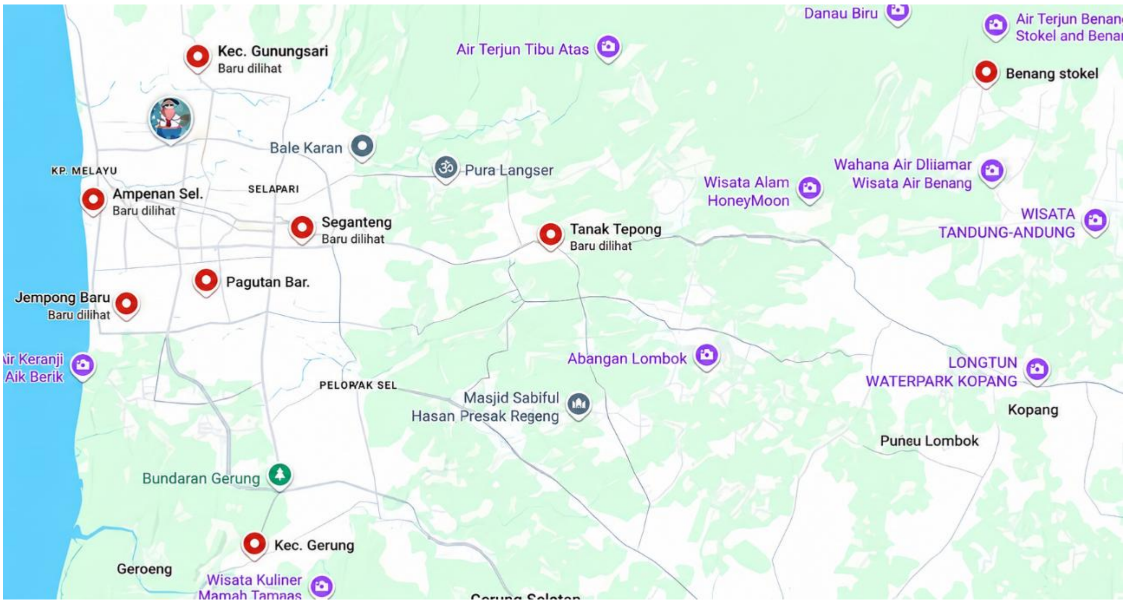

Fig 2. Map of production centers and delivery points

Based on the distribution locations that have been identified, a weighted graph is then formed that represents the relationships between distribution locations. The graph is the basis for the preparation of the weight matrix used in the Min-Plus Algebra calculation process.

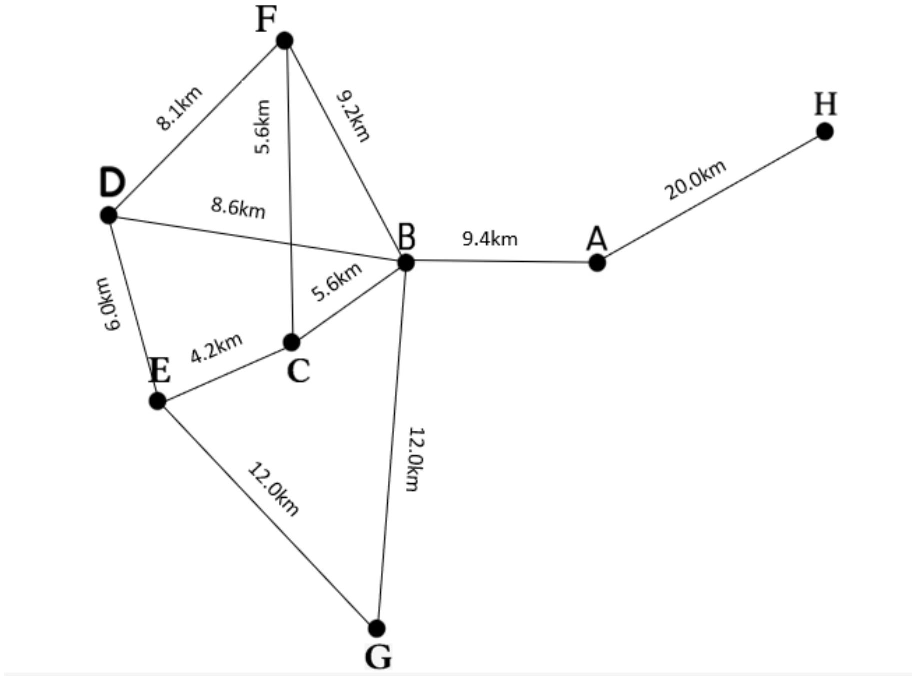

Fig 3. Weighted directionless graph representation of distribution networks

Remarks:

A : Manufacturer's Starting Point (Tanak tepong)

B : Seganteng

C : Pagutan
D : Ampenan
E : Jempong
F : Gunung Sari
G : Gerung
H : Benang Stokel

Based on the graph, node A represents the production center, while nodes B to H represent the distribution points in the surrounding area. Each side (edge) shows a direct connection between locations with a weight in the form of actual mileage. This graph representation provides a structured picture of the relationship between points and forms the basis for the preparation of the distance weight matrix at the next stage of analysis.

3.3 Weight Matrix Formation

Once the distribution network is represented in the form of a weighted graph, the next step is to form a weight matrix that will be used in the Min-Plus Algebra calculation process. The weight matrix is arranged based on the distance between distribution locations obtained from Google Maps and is represented in the form of a square matrix measuring 8×8 according to the number of nodes on the distribution network.

In the weight matrix, each element (a_{ji}) expresses the distance from node (j) to node (i). If there is a direct relationship between two nodes, then the value of the matrix element is populated with the actual distance between the locations. Conversely, if there is no direct relationship, then the matrix element is given an infinite value (∞) or (ε) in Min-Plus Algebra. The distribution network weight matrix is shown in Table 1.

Table 1. Matrix Weight A Distribution Network

From/To	A	B	C	D	E	F	G	H
A	ε	9. 4	ε	ε	ε	ε	ε	20.0
B	9. 4	ε	5.6	8.6	ε	9.2	12.0	ε
C	ε	5.6	ε	ε	4.2	5.6	ε	ε
D	ε	8.6	ε	ε	6.0	8.1	ε	ε
E	ε	ε	4.2	6.0	ε	ε	12.0	ε
F	ε	9.2	5.6	8.1	ε	ε	ε	ε
G	ε	12.0	ε	ε	12.0	ε	ε	ε
H	20.0	ε	ε	ε	ε	ε	ε	ε

Based on the weight matrix that has been formed, it can be known that the direct relationship between distribution locations and the distance that connects them can be determined. This matrix is the basis of the Min-Plus Algebra operation process because all minimum trajectory calculations are carried out based on the weight information contained in the matrix. Furthermore, the weight matrix is used in the Min-Plus Algebra operation process through matrix rank to obtain the minimum distance between all node pairs on the distribution network.

3.4 Application of Min-Plus Algebra and Closure Matrix Formation A^*

After the weight matrix A is constructed, Min-Plus Algebra is employed to determine the minimum distance between all pairs of nodes in the distribution network. The computation is carried out through repeated Min-Plus matrix exponentiation until the closure matrix A^* is obtained. The resulting closure matrix provides the minimum distance between every pair of distribution locations by considering all feasible paths within the network.

Based on Equations (1) through (3), matrix exponentiation is performed using Min-Plus multiplication to obtain the minimum distance between nodes. The calculation begins by determining the matrix $A^{\otimes 2}$. As an illustration, one of the elements of matrix $A^{\otimes 2}$is calculated as follows:

Determining element $(A^{\otimes 2})_{13}$.

$$(A^{\otimes 2})_{13} = \oplus_{k=1}^{8} (a_{1k} \otimes a_{k3})$$

In Min-Plus Algebra:

$$\oplus = Min\ (-) \text{ and } \otimes = Plus\ (+)$$

Thus,

$$(A^{\otimes 2})_{13} = min\{a_{11} + a_{13}, a_{12} + a_{23}, a_{13} + a_{33}, a_{14} + a_{43}, a_{15} + a_{53}, a_{16} + a_{63}, a_{17} + a_{73}, a_{18} + a_{83}\}$$

Substitute the values from matrix A from Table 1:

$$(A^{\otimes 2})_{13} = \{\varepsilon + \varepsilon, 9.4 + 5.6, \varepsilon + \varepsilon, \varepsilon + \varepsilon, \varepsilon + 4.2, \varepsilon + 5.6, \varepsilon + \varepsilon, 20.0 + \varepsilon\}$$

Since:

$$\varepsilon + x = \varepsilon$$

Then,

$$(A^{\otimes 2})_{13} = min\{\varepsilon, 15.0, \varepsilon, \varepsilon, \varepsilon, \varepsilon, \varepsilon, \varepsilon\}$$

Therefore:

$$(A^{\otimes 2})_{13} = 15.0$$

Thus, the value of the element in the first column and third row of matrix $A^{\otimes 2}$ is 15.0.

Using the same procedure, the remaining elements are calculated to obtain matrix $A^{\otimes 2}$. The resulting matrix is presented as follows.

$$A^{\otimes 2} = \begin{bmatrix} 18.8 & \varepsilon & 15.0 & 18.0 & \varepsilon & 18.6 & 21.4 & \varepsilon \\ \varepsilon & 11.2 & 14.8 & 17.3 & 9.8 & 11.2 & \varepsilon & 29.4 \\ 15.0 & 14.8 & 8.4 & 10.2 & \varepsilon & 14.8 & 16.2 & \varepsilon \\ 18.0 & 17.3 & 10.2 & 12.0 & \varepsilon & 17.8 & 18.0 & \varepsilon \\ \varepsilon & 9.8 & \varepsilon & \varepsilon & 8.4 & 9.8 & \varepsilon & \varepsilon \\ 18.6 & 11.2 & 14.8 & 17.8 & 9.8 & 11.2 & 21.2 & \varepsilon \\ 21.4 & \varepsilon & 16.2 & 18.0 & \varepsilon & 21.2 & 24.0 & \varepsilon \\ \varepsilon & 29.4 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & 40.0 \end{bmatrix}$$

Next, matrix $A^{\otimes 3}$ obtained by the Min-Plus multiplication operation between the matrix $A^{\otimes 2}$ and matrix A. The calculation process is carried out by the same procedure as in the formation of the matrix $A^{\otimes 2}$.

$$\begin{aligned} (A^{\otimes 3})_{12} &= \oplus_{k=1}^{8} (a_{1k} \otimes a_{k2}) \\ &= min\{a_{11} + a_{12}, a_{12} + a_{22}, a_{13} + a_{32}, a_{14} + a_{42}, a_{15} + a_{52}, a_{16} + a_{62}, a_{17} + a_{72}, a_{18} + a_{82}\} \\ &= min\{18.8 + 9.4, \varepsilon + \varepsilon, 15.0 + 5.6, 18.0 + 8.6, \varepsilon + \varepsilon, 18.6 + 9.2, 21.4 + 12.0, \varepsilon + \varepsilon\} \\ &= min\{28.2, \varepsilon, 20.6, 26.6, \varepsilon, 27.8, 33.4, \varepsilon\} \\ &= 20.6 \end{aligned}$$

Using the same procedure, the remaining elements are calculated to obtain matrix $A^{\otimes 3}$. The resulting matrix is presented as follows.

$$A^{\otimes 3} = \begin{bmatrix} \varepsilon & 20.6 & 24.2 & 26.6 & 19.2 & 20.6 & \varepsilon & 38.8 \\ 20.6 & 20.4 & 14.0 & 15.8 & 19.0 & 20.4 & 21.8 & \varepsilon \\ 24.2 & 14.0 & 20.4 & 22.9 & 12.6 & 14.0 & 26.8 & 35.0 \\ 26.6 & 15.8 & 22.9 & 25.9 & 14.4 & 15.8 & 29.3 & 38.0 \\ 19.2 & 19.0 & 12.6 & 14.4 & \varepsilon & 19.0 & 20.4 & \varepsilon \\ 20.6 & 20.4 & 14.0 & 15.8 & 19.0 & 20.4 & 21.8 & 38.6 \\ \varepsilon & 21.8 & 26.8 & 29.3 & 20.4 & 21.8 & \varepsilon & 41.4 \\ 38.8 & \varepsilon & 35.0 & 38.0 & \varepsilon & 38.6 & 41.4 & \varepsilon \end{bmatrix}$$

To avoid presenting repetitive calculations, only the formation of matrices $A^{\otimes 2}$ and $A^{\otimes 3}$ is shown in this study. The same computational procedure is then continued until $A^{\otimes 7}$. Since the network contains eight nodes, the matrix power operation is performed up to $A^{\otimes (n-1)}$. This allows the minimum distances between all pairs of nodes to be determined through the Min-Plus Algebra framework.

After the entire matrix power computation process is completed, the closure matrix A^* is constructed based on Equation 5 as follows:

$$A^* = I \oplus A \oplus A^{\otimes 2} \oplus A^{\otimes 3} \oplus \cdots \oplus A^{\otimes 7}$$

where I is the identity matrix in Min-Plus Algebra and n denotes the number of nodes in the distribution network. Since the network consists of eight nodes, the closure matrix is obtained by combining the matrix powers up to $A^{\otimes(7)}$.

The resulting closure matrix A^*represents the minimum distances between all pairs of nodes in the network. Each element of A^*corresponds to the shortest distance that can be achieved between two nodes by considering all possible paths within the distribution network.

$$A^* = \begin{bmatrix} \varepsilon & \mathbf{9.4} & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ \mathbf{9.4} & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \mathbf{9.2} & \mathbf{12.0} & \varepsilon \\ 41.0 & \mathbf{5.6} & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ 42.8 & 32.6 & \varepsilon & \varepsilon & \mathbf{6.0} & \varepsilon & \varepsilon & \varepsilon \\ 36.0 & \varepsilon & \mathbf{4.2} & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ 37.4 & 37.2 & 30.8 & \mathbf{8.1} & \varepsilon & \varepsilon & \varepsilon & \varepsilon \\ 21.4 & \mathbf{12.0} & 16.2 & 18.0 & 37.2 & 21.2 & \varepsilon & 41.4 \\ \mathbf{20.0} & 29.4 & 35.0 & 38.0 & 39.2 & 38.6 & 41.4 & \varepsilon \end{bmatrix}$$

The closure matrix A^* represents the minimum distance between all pairs of nodes in the distribution network by considering all possible paths. Each element in A^*indicates the shortest distance from one distribution location to another, including indirect routes through intermediate nodes. From the obtained closure matrix, it can be observed that several indirect paths result in shorter distances than the corresponding direct connections in the initial weight matrix. This demonstrates the ability of Min-Plus Algebra to identify shortest-path relationships within the distribution network.

Therefore, the closure matrix A^* is used as the basis for analyzing the distribution network structure and deriving a distribution route, which is discussed in the following section.

3.5 Determination of the Distribution Route

Based on the manual computation using Min-Plus Algebra, a closure matrix (A^*) was obtained, containing the minimum distances between each pair of locations in the distribution network. This matrix represents the shortest distance that can be traveled from one node to another.

Based on the analysis of the closure matrix (A^*) and the structure of the distribution network, the following distribution route was identified

$$A \to B \to C \to F \to D \to E \to G \to B \to A \to H.$$

The total distance of this route is 88.1 km, calculated by summing the distances between consecutive locations along the route. The closure matrix (A^*) is used to determine the minimum distances between connected nodes, while the route was arranged by considering the connectivity of the distribution network so that all distribution locations could be reached.

The results indicate that Min-Plus Algebra can be applied to analyze shortest-path relationships within a distribution network and support the determination of distribution routes based on the existing network structure. The obtained route provides a representation of the distribution path connecting all locations in the UMKM Putra-Putri Agli distribution network.

3.6 Visualization of Results

Based on the results of the Min-Plus Algebra computation, the closure matrix A^* was obtained, containing the minimum distances between each pair of nodes in the distribution network. Each element a_{ji} represents the shortest distance from node j to node i. To provide a clearer representation of the network structure derived from the closure matrix A^*, the results are visualized in the form of a graph showing the relationships among distribution nodes and the shortest-path connections within the network.

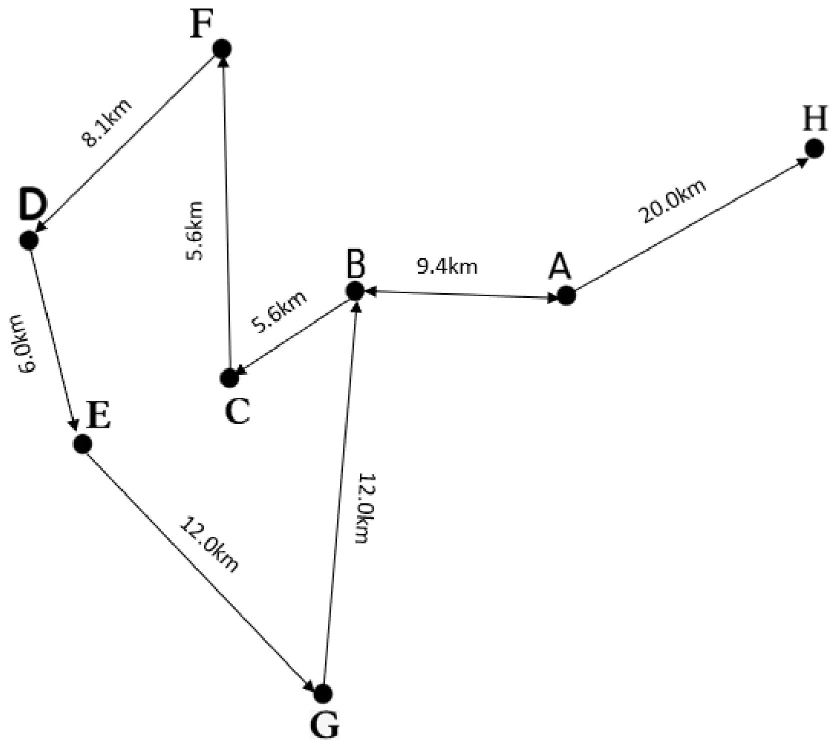

Fig 4. Graph representation of the distribution network

The resulting graph is used to illustrate the distribution route obtained from the network analysis, namely:

$$A \rightarrow B \rightarrow C \rightarrow F \rightarrow D \rightarrow E \rightarrow G \rightarrow B \rightarrow A \rightarrow H$$

with a total distance of 88.1 km. This visualization shows the traversal pattern within the distribution network, where certain nodes are visited more than once due to their role as connecting points between different locations. The repeated visits indicate the limited availability of alternative direct paths, requiring the route to pass through intermediate nodes to reach all distribution locations.

The graphical representation highlights the influence of the distribution network structure on route formation. It demonstrates that the resulting route is determined not only by the minimum distances between node pairs contained in A^*, but also by the connectivity constraints of the network. Therefore, the route reflects the structure of the existing distribution network while incorporating the shortest-path information obtained through Min-Plus Algebra.

4 Conclusion

This study analyzes the distribution network of UMKM Putra-Putri Agli using Min-Plus Algebra to determine the shortest path structure within a weighted graph representation. The results show that the topology of the distribution network significantly influences route formation, where limited connectivity between nodes requires the use of intermediate nodes to reach minimum-distance paths. This indicates that distribution efficiency is determined not only by direct distances but also by the structural configuration of the network.

The application of Min-Plus Algebra provides a systematic matrix-based framework for solving the all-pairs shortest path problem through the construction of the closure matrix A^*. The matrix A^* contains the minimum distance values between all pairs of nodes and serves as the basis for identifying shortest-path relationships within the distribution network. Through this approach, the shortest distances between locations can be obtained simultaneously within a unified algebraic framework, making it suitable for network analysis and route planning.

From a practical perspective, the results provide a structured reference for distribution planning in UMKM Putra-Putri Agli. The identification of minimum-distance paths can support more efficient delivery planning by reducing unnecessary travel distances and improving operational flow. However, this study is limited to static distance-based analysis and does not consider dynamic factors such as traffic conditions, travel time variations, or vehicle capacity constraints. Therefore, future research is recommended to incorporate dynamic network parameters or integrate Min-Plus Algebra with other optimization methods to improve its applicability in real-world distribution systems.

References

[1] A. Rahmana and Suseno, 'Usulan Rute Distribusi Terpendek Menggunakan Metode Saving Matrix untuk Minimasi Jarak dan Biaya Distribusi', *Jurnal Ilmiah Research and Development Student*, vol. 3, no. 1, pp. 72–84, 2025, doi: 10.59024/jis.v3i1.1048.

[2] N. A. M. Nordin, S. S. R. Shariff, S. S. Supadi, and Y. Pahala, 'Determining the Shortest Path for Inner Warehouse Transportation', *Journal of ASIAN Behavioural Studies*, vol. 8, no. 25, pp. 1–21, 2023, doi: 10.21834/jabs.v8i25.425.

[3] Z. Wei, 'High Performance Computing Simulation of Intelligent Logistics Management Based on Shortest Path Algorithm', *Sci. Program.*, vol. 2022, pp. 1–10, 2022, doi: 10.1155/2022/7930553.

[4] K. Arman and A. Tuş, 'Applying the Node Combination Algorithm to the Shortest Path Problem for a Logistics Firm', *Journal of Transportation and Logistics*, vol. 7, no. 2, pp. 289–302, 2022, doi: 10.26650/JTL.2022.1092188.

[5] I. Muyamina and A. Safira, 'Implementation of the Shortest Path Method with Excel Solver to Optimize Goods Delivery Routes', *Journal of Computer Science Advancements*, vol. 2, no. 1, pp. 27–32, 2024, doi: 10.70177/jcsa.v2i1.1137.

[6] J. Guo, H. Liu, T. Liu, G. Song, and B. Guo, 'The Multi-Objective Shortest Path Problem with Multimodal Transportation for Emergency Logistics', *Mathematics*, vol. 12, no. 17, pp. 1–25, 2024, doi: 10.3390/math12172615.

[7] Z. Niu, S. Wu, and X. Zhou, 'Efficient Mathematical Lower Bounds for City Logistics Distribution Network with Intra-Echelon Connection of Facilities: Bridging the Gap from Theoretical Model Formulations to Practical Solutions', *Algorithms*, vol. 16, no. 5, May 2023, doi: 10.3390/a16050252.

[8] E. Y. He, N. Boland, G. Nemhauser, and M. Savelsbergh, 'Dynamic Discretization Discovery Algorithms for Time-Dependent Shortest Path Problems', *INFORMS J. Comput.*, vol. 34, no. 2, pp. 1086–1114, 2022, doi: 10.1287/ijoc.2021.1084.

[9] R. Jaballah, M. Veenstra, L. C. Coelho, and J. Renaud, 'The Time-Dependent Shortest Path and Vehicle Routing Problem', *INFOR: Information Systems and Operational Research*, vol. 59, no. 4, pp. 592–622, 2021, doi: 10.1080/03155986.2021.1973785.

[10] N. Farhi, 'A Min-Plus Algebra System Theory for Traffic Networks', *Mathematics*, vol. 11, no. 19, pp. 1–41, Oct. 2023, doi: 10.3390/math11194028.

[11] M. Joswig and B. Schröter, 'Parametric Shortest-Path Algorithms via Tropical Geometry', *Mathematics of Operations Research*, vol. 47, no. 3, pp. 2065–2081, 2022, doi: 10.1287/moor.2021.1199.

[12] B. Masing, N. Lindner, and E. Bortoletto, 'EURO Journal on Transportation and Logistics Computing all shortest passenger routes with a tropical Dijkstra algorithm', *EURO Journal on Transportation and Logistics*, vol. 14, no. December 2024, p. 100163, 2025, doi: 10.1016/j.ejtl.2025.100163.

[13] T. K. Bhatia, A. Kumar, S. S. Appadoo, Y. Gajpal, and M. K. Sharma, 'Mehar Approach for Finding Shortest Path in Supply Chain Network', *Sustainability*, vol. 13, no. 7, p. 19, 2021, doi: 10.3390/su13074016.

[14] G. Zuzic, G. Goranci, M. Ye, B. Haeupler, and X. Sun, 'Universally-Optimal Distributed Shortest Paths and Transshipment via Graph-Based ℓ1-Oblivious Routing', in *Proceedings of the Annual ACM-SIAM Symposium on Discrete Algorithms (SODA)*, SIAM, 2022, pp. 2549–2579. doi: 10.1137/1.9781611977073.100.

[15] W. Lee, W. Li, B. Lin, and A. Monod, 'Tropical optimal transport and Wasserstein distances', *Information Geometry*, vol. 5, no. 1, pp. 247–287, 2022, doi: 10.1007/s41884-021-00046-6.