Mathematical Journal of Modelling and Forecasting
Vol. 4, No. 1, June 2026, pp. 20-26

Analysis of the Application of Queue Theory in Ambacang Community Health Centers Padang

Sukra Hamna[1,*], and Nadiyatul Ghina[2]

[1,2] Department of Mathematics, Universitas Negeri Padang, Indonesia
*sukrahamna07@gmail.com

Abstract. Community health centers (Puskesmas) play a crucial role in providing primary healthcare services. However, long patient queues often reduce service efficiency and patient satisfaction. This study analyzes queue performance at a Puskesmas outpatient clinic using the M/M/1 queuing model to evaluate waiting time and service effectiveness. Data were collected through seven days of field observations, recording patient arrival patterns and service times during operational hours. The results show that the patient arrival rate is 0.2190 patients per minute, while the service rate is 0.2944 patients per minute. The system operates under a stable condition with a utilization factor of 0.744. The average waiting time in the queue is 9.87 minutes, whereas the total time spent in the system is 13.27 minutes, indicating that a significant proportion of patient time is spent waiting for service. The originality of this study lies in its empirical application of the M/M/1 queuing model to a real outpatient healthcare system in a Puskesmas setting, providing practical performance indicators that can support operational decision-making in primary healthcare services. However, the study is limited by a relatively short observation period of seven days and data collected from a single healthcare facility, meaning the findings reflect local conditions and cannot be generalized to other healthcare centers with different operational characteristics. The study recommends improving workforce management and implementing a structured patient scheduling system to reduce waiting time and enhance service efficiency.

Keywords: Queueing Theory, Community Health Center, Waiting Time.

1 Introduction

A Community Health Center (Puskesmas) is a primary healthcare facility that serves as a medical care provider for patients and the public, prioritizing preventive measures and health education to achieve the highest possible level of public health within its area of responsibility [1]. In its role as an operational implementing agency under the auspices of the regional health authority, Puskesmas has a vision and mission to provide the best medical services for the welfare of the community in its working area [2]. Nevertheless, Puskesmas still faces a major challenge in the form of long patient waiting times spanning from the initial registration phase and medical treatment to pharmacy services a situation that can negatively impact patient satisfaction [3]. To achieve its overarching goals, Puskesmas must continuously improve the quality of patient care, one of which is by optimizing the queuing system to reduce waiting times and enhance operational efficiency. An effective and efficient queuing system is crucial to ensure the smooth running of healthcare services, as long queues can diminish patient satisfaction and potentially increase the risk of disease transmission [4].

A system can be defined as a unified entity formed by integrated parts or components that collaborate systematically to achieve a specific goal [5]. A queue is a waiting line of patients or units requiring attention from one or more service providers. This condition occurs when the demand for services exceeds the existing operational capacity, forcing arriving patients to wait their turn because the service facility is currently busy [6]. Queuing theory plays a critical role in decision-making regarding the determination of resources needed to achieve optimal service by examining system components such as arrival patterns, service delivery mechanisms, the number of provider facilities, and overall operational capacity [7]. The literature on queuing theory indicates that mathematical modeling can be used to understand the behavior of service systems, particularly under conditions where demand exceeds capacity [8].

To improve the quality of healthcare services at Puskesmas, a quantitative approach capable of comprehensively evaluating the system is required. One widely used method is queuing theory, specifically the M/M/1 model approach, which illustrates an operational mechanism through a single service channel with random arrival and service distributions [9]. The application of this model enables Puskesmas management to analyze operational performance and determine the utilization rate of service facilities [10].

With this information, strategic measures such as the addition of medical personnel, adjustment of service schedules, or the implementation of digital queuing systems can be formulated more accurately. Furthermore, the integration of queuing theory with simulation methods, as demonstrated in a study at Puskesmas Karang Joang, has proven to yield precise and reliable results as a foundation for policymaking in service management [11]. In addition to highlighting technical aspects, it is also essential to consider patient perception, as long waiting times can diminish the satisfaction of service users [12]. Thus, the application of an appropriate queuing model can serve as a strategic solution to optimize service performance and patients comfort levels at community health centers.

2 Research Methods

This study is classified as applied research that utilizes primary data factually collected at the Puskesmas (Community Health Center). The acquired information encompasses the number of patient arrivals along with the duration of service within a specific time period. The data processing was executed through the following stages:

1. Conducting descriptive data analysis by presenting a graph of the average patient arrivals, as well as collecting data on the number of patient arrivals and service durations at the Puskesmas over a specified period.
2. Analyzing the queuing system to develop system performance measures using the following procedures:
   a. Calculating the average number of arrivals (λ) using the formula:
   $$\lambda = \frac{number\ of\ patients}{long\ observation\ hours} \tag{1}$$
   b. Calculating the average service duration (μ) using the equation:
   $$\mu = \frac{1}{\frac{total\ service\ time}{n}} \tag{2}$$
   Subsequently, identifying the data to fulfill the steady-state condition $\left(\rho = \frac{\lambda}{c\mu} < 1\right)$ [13].
3. Performing a Chi-Square Test to examine the goodness-of-fit for the arrival and service distributions with the queuing model [14].
4. Analyzing the characteristics and the M/M/1 queuing system model using the following procedures [15]:
   a. Calculating the probability that the service facility is idle (P_0) using the equation:
   $$P_0 = 1 - \frac{\lambda}{\mu} \tag{3}$$
   b. Calculating the average number of patients in the entire service system (L) using the equation:
   $$L = \frac{\lambda}{\mu - \lambda} \tag{4}$$
   c. Calculating the average number of patients in the queue (L_q) using the equation:
   $$L_q = \frac{\lambda^2}{\mu(\mu - \lambda)} \tag{5}$$
   d. Calculating the average waiting time in the system (W) using the equation:
   $$W = \frac{1}{\mu - \lambda} \tag{6}$$
   e. Calculating the average waiting time to receive medical service in the queue (W_q) using the equation:
   $$W_q = \frac{\lambda}{\mu(\mu - \lambda)} \tag{7}$$
5. Drawing conclusions regarding the service quality at the Puskesmas based on the research and analysis results.

3 Result and Discussion

3.1 Data Description

The primary data utilized encompass the number of patient arrivals and service durations at a healthcare facility in Padang City. The data were collected daily with an observation period of 3 hours, specifically from 9:00 AM to 12:00 PM. The average patient arrivals over 7 days at 1-hour intervals are presented in Figure 1.

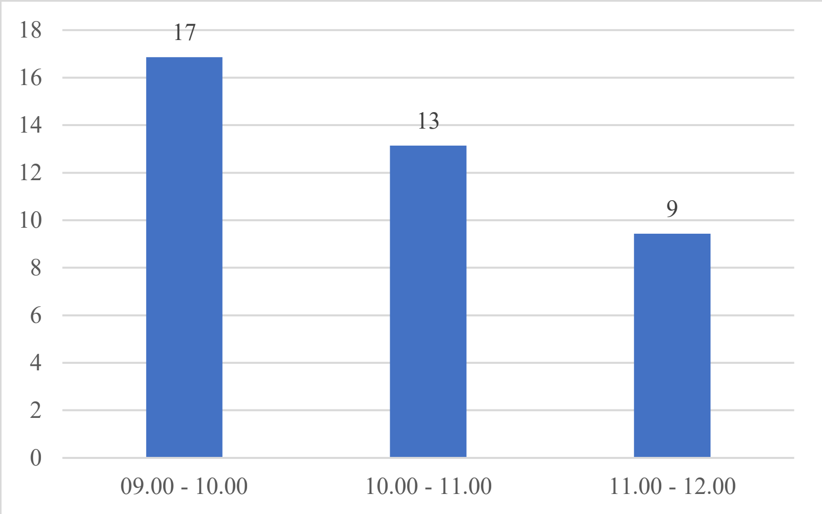

Fig 1. Average Patient Arrivals at 1-Hour Intervals

Based on Fig 1, the estimated average patient arrivals reached their peak between 09:00 and 10:00 AM. The arrival rate and medical service duration were subsequently applied as the basis for calculations, referring to these peak time conditions. This approach is predicated on the fact that patient volume increases during peak service periods; thus, if not managed properly, it can lead to congestion and increasingly longer queues.

3.2 Data Analysis

Data Collection of Patient Arrivals and Service Rates

A descriptive statistical analysis of the patient arrival data and service durations at the Puskesmas during the 09:00 – 12:00 period over a 7-day timeframe is presented in Table 1 below.

Table 1. Recapitulation of the quantity of patient arrivals and the estimated average duration of medical services

No	Day	Time Period (Hours)	Patient Arrivals (λ)	Average Patient Service Time (μ)
1	Monday	09.00 - 10.00	20	3.09
		10.00 - 11.00	15	3.56
		11.00 - 12.00	12	3.35
2	Tuesday	09.00 - 10.00	21	2.49
		10.00 - 11.00	14	3.39
		11.00 - 12.00	8	3.45
3	Wednesday	09.00 - 10.00	17	3.28
		10.00 - 11.00	13	4.14
		11.00 - 12.00	9	3.47
4	Thursday	09.00 - 10.00	13	3.45
		10.00 - 11.00	14	3.39

No	Day	Time Period (Hours)	Patient Arrivals (λ)	Average Patient Service Time (μ)
		11.00 - 12.00	8	3.45
5	Friday	09.00 - 10.00	14	3.17
		10.00 - 11.00	10	4.42
		11.00 - 12.00	8	5.07
6	Saturday	09.00 - 10.00	16	2.45
		10.00 - 11.00	12	3.40
		11.00 - 12.00	9	4.20
7	Monday	09.00 - 10.00	17	2.39
		10.00 - 11.00	14	3.00
		11.00 - 12.00	12	3.20
Total			276	71.33

The data indicate that the number of patient arrivals varies hourly and daily, with a total of 276 patients during the 09:00–12:00 period. The average service time also differs, ranging from 2.39 to 5.07 minutes per patient. Peak hours typically occur between 09:00 and 10:00, whereas service times tend to be longer between 11:00 and 12:00. It is crucial to consider these variations to improve service efficiency and the queuing system at the Puskesmas. It should be noted that the data used in this study were collected over a seven-day observation period at a single Puskesmas. Therefore, the findings reflect the operational characteristics of the observed healthcare facility and should not be generalized to other Puskesmas or healthcare institutions with different service capacities, patient volumes, and operational procedures.

Steady-State Analysis

1. Average Patient Arrivals

Based on Equation (1), the average patient arrival rate is obtained as follows:

$$\lambda = 0.2190 \text{ patient/minute}$$

This result indicates that, on average, approximately 0.2190 patients arrive at the clinic every minute, which is equivalent to about 13.14 patients per hour. This value represents the average demand for healthcare services during the observation period.

2. Average Patient Service Time

Based on Equation (2), the average patient service rate (μ) is obtained as follows:

$$\mu = 0.2944 \text{ patient/minute}$$

This result indicates that the general practitioner is able to serve approximately 0.2944 patients per minute, or about 17.66 patients per hour. Since the service rate is higher than the arrival rate ($\mu > \lambda$), the service facility has sufficient capacity to accommodate incoming patients during the observation period.

Referring to the information obtained from field observations over a full week at a Puskesmas Ambacang Padang, the value of ρ (the utilization rate or the probability that the medical facility is busy) was determined.

$$\rho = \frac{\lambda}{c\mu} = \frac{0.2190}{1(0.2944)} = 0.7440$$

Because $\rho < 1$, the system is considered to be in a steady-state phase. This finding confirms that the arrival rate of patients is lower than the capacity of patients that can be served.

Data Distribution Testing

1. Chi-Square Test for Patient Arrivals

The Chi-Square testing method was applied to examine the goodness-of-fit between the patient arrival data distribution and the Poisson distribution. This test is essential to ensure that patient arrival patterns can be mathematically modeled using the appropriate distribution. The proposed hypotheses are:

H_0 : The patient arrival data follows a Poisson distribution.
H_1 : The patient arrival data does not follow a Poisson distribution.

The analysis results demonstrate a calculated Chi-Square value $\chi^2_{calculated}$ is 3.2022. Based on the Chi-Square distribution table at a significance level of $\alpha = 0.05$, the tabulated Chi-Square value χ^2_{table} is 3.841. Because $\chi^2_{calculated} < \chi^2_{table}$, the null hypothesis H_0 is accepted. From this explanation, it can be concluded that the recapitulation of patient arrival data at the community health center follows a Poisson distribution.

2. Chi-Square Test for Medical Service Duration

Furthermore, the Chi-Square evaluation method was also conducted to test whether the duration of medical treatment follows an exponential distribution pattern, as is commonly used in queuing theory to represent inter-arrival times and service duration. The proposed hypotheses are:

H_0 : The duration of patient medical treatment aligns with an exponential distribution.
H_1 : The duration of patient medical treatment does not fit an exponential distribution pattern.

From the data processing results, the calculated Chi-Square value $\chi^2_{calculated}$ is 2.3318. The tabulated value χ^2_{table} at $\alpha = 0.05$ is 5.991. Since $\chi^2_{calculated} < \chi^2_{table}$, the null hypothesis H_0 is not rejected. In other words, patient service times at the community health center can be assumed to follow an exponential distribution.

Justification of M/M/1 Assumptions

The M/M/1 queuing model was selected based on the characteristics of the service system observed at the General Practitioner Clinic of the Puskesmas. Patient arrivals were assumed to follow a Poisson distribution, while service times were assumed to follow an exponential distribution, as confirmed by the Chi-Square goodness-of-fit tests. The single-server assumption was satisfied because only one general practitioner provided services during the observation period. In addition, patients were generally served according to the First-In-First-Out (FIFO) discipline, and no priority-based service mechanism was observed. The assumptions of an infinite calling population and unlimited queue capacity were adopted as standard approximations in queuing theory. Based on these service characteristics and statistical test results, the M/M/1 model was considered appropriate for analyzing the queuing system at the Puskesmas Ambacang.

Queuing Theory Analysis

1. The probability that the system is empty is calculated using Equation (3).

$$P_0 = 1 - \frac{\lambda}{\mu} = 0.2559$$

   Thus, the obtained value of indicates that there is a probability of 0.2559 that there are no patients in the system, meaning the service facility is currently idle.
2. The average number of patients in the entire service facility is calculated using Equation (4).

$$L = \frac{\lambda}{\mu - \lambda} = 2.9067$$

   Based on the calculation, L is obtained as 2.9067, which means that, on average, there are approximately 3 patients within the system, either currently being served or waiting.
3. The average number of patients in the queue is calculated using Equation (5).

$$L_q = \frac{\lambda^2}{\mu(\mu - \lambda)} = 2.1627$$

From the above calculation, L_q is obtained as 2.1627, indicating that a significant portion of patients are in a waiting state prior to receiving service.

4. The average waiting time in the system is calculated using Equation (6).

$$W = \frac{1}{\mu - \lambda} = 13.2698$$

The result for the average time in the system is $W = 13.2698$, meaning patients spend approximately 13.27 minutes from arrival until the completion of service.

5. The average waiting time to receive medical service in the queue is calculated using Equation (7).

$$W_q = \frac{\lambda}{\mu(\mu - \lambda)} = 9.8732$$

From the calculation results, W_q is obtained as 9.8732 minutes. This indicates that a portion of the patient's time is spent waiting before receiving service.

Limitation of Study

The limitations of this study are as follows:

1. The analysis was based on data collected over a relatively short observation period of seven days. Consequently, the data may not fully capture long-term variations in patient arrival patterns and service times.
2. The study was conducted at a single Puskesmas and focused on the General Practitioner Clinic. Therefore, the findings reflect the local characteristics of the observed service system.
3. The results should not be generalized to other healthcare facilities, as differences in patient volume, staffing levels, service procedures, and operational policies may affect queueing performance.
4. The study employed the M/M/1 queuing model based on the observed service conditions. Different queueing models may be more appropriate for healthcare facilities with multiple service channels or more complex service structures.

4 Conclusion

Based on the analysis of the M/M/1 queuing model at the Puskesmas polyclinic, an average patient arrival rate (λ) of 0.2190 patients/minute and a service rate (μ) of 0.2944 patients/minute indicate that the system is in a stable condition $(\rho < 1)$. However, the relatively long waiting time in the queue (9.87 minutes) signifies the need for efficiency improvements, such as staff training or patient schedule management. This condition demonstrates that although the system is operating stably, service efficiency still needs to be enhanced to minimize patient waiting times. Several efforts that can be undertaken include improving staff performance through training, increasing the number of service personnel during peak hours, and organizing patient arrival schedules to be more evenly distributed. With these improvements, it is expected that the service quality at the Puskesmas will increase, leading to improved patient satisfaction.

References

[1] Kementerian Kesehatan Republik Indonesia, "Peraturan Menteri Kesehatan Republik Indonesia Nomor 75 Tahun 2014 tentang Pusat Kesehatan Masyarakat," 2014.

[2] Kementerian Kesehatan Republik Indonesia, "Pedoman Pelaksanaan Promosi Kesehatan di Daerah," Jakarta, 2010.

[3] H. Milaenia Fitri and M. Hidayati, "Pengaruh Lama Waktu Tunggu Pendaftaran Terhadap Kepuasan Pasien di Puskesmas Waringinkurung," *Cerdika: Jurnal Ilmiah Indonesia*, vol. 1, no. 12, pp. 1789–1795, Dec. 2021, doi: 10.36418/cerdika.v1i12.262.

[4] N. V. Amalia *et al.*, "Analisis Optimasi Sistem Antrian Poli Umum Puskesmas Wonopringgo Pekalongan," *Konsorsium Seminar Nasional Waluyo Jatmiko*, vol. 16, pp. 611–620, Nov. 2023, doi: 10.33005/wj.v16i1.11.

[5] H. M. Jogiyanto, *Analisis dan Disain: Sistem Informasi: Pendekatan Terstruktur Teori dan Praktek Aplikasi Bisnis*. Yogyakarta: Andi Offset, 1990.

[6] E. Rosellawati, “Evaluasi Sistem Antrian Pelayanan Pasien pada Puskesmas di Wonosobo,” Universitas Islam Indonesia, Yogyakarta, 2018.

[7] G. Khoirunnisa and N. Martini, “Analisis Sistem Antrian di Dinas Kependudukan dan Pencatatan Sipil Kabupaten Karawang,” *Jurnal Ilmiah Ekonomi Global Masa Kini*, vol. 12, 2021.

[8] W. Trianto, E. A. Firdaus, and B. A. Suburdjati, “Analisis Sistem Antrian Pendaftaran Menggunakan Metode Queuing System di Puskesmas Kota Cimahi,” *Jurnal Nuansa Informatika*, vol. 15, no. 2, 2021, [Online]. Available: https://journal.uniku.ac.id/index.php/ilkom

[9] A. M. S and A. Ahdiat, “Perancangan dan Implementasi Model Sistem Antrian Pelayanan di Puskesmas Mulya Mekar,” *Majalah Ilmiah Solusi Unsika*, vol. 10, 2012.

[10] S. Alimuddin and M. Ahsan, “Analisis Sistem Antrian dan Optimalisasi Layananan pada UPTD Puskesmas Lakessi Parepare,” *Journal of Mathematics Learning Innovation (JMLI)*, vol. 1, no. 2, pp. 163–175, 2022.

[11] M. K. Fajar and A. Jamil, “Simulasi Sistem Antrian Pelayanan pada Puskesmas Karang Joang KM. 12 Balikpapan Utara,” *Conference on Electrical Engineering, Informatics, Industrial Technology, and Creative Media*, vol. 4, no. 1, 2024.

[12] M. O. Dewi and E. Marsepa, “Pengaruh Waktu Tunggu terhadap Kepuasan Pasien di Puskesmas Mauk Kabupaten Tangerang Banten,” *Nusantara Hasana Journal*, vol. 1, no. 7, pp. 33–37, 2021.

[13] M. N. Khasanah and Y. P. Astuti, “Analisis Sistem Antrian pada Optimalisasi Pelayanan Pasien di Pusat Kesehatan Masyarakat,” *MATHunesa: Jurnal Ilmiah Matematika*, vol. 10, no. 1, 2022.

[14] S. Arina, R. Harahap, U. Sinulingga, and S. Ariswoyo, “Analisis Sistem Antrian Pelayanan Nasabah di PT. Bank Negara Indonesia (Persero) Tbk Kantor Cabang Utama USU,” *Saintia Matematika*, vol. 2, pp. 277–287, 2014.

[15] Siti Khadijah Laoly, Wilia Usna, Nurhalimah Ritonga, and Hendra Cipta, “Implementasi Teori Antrian Single Chanel Model (M/M/1) Menggunakan Microsoft Excel Dalam Pengujian Kelayakan Kendaraan Bermotor Pada Dinas Perhubungan Kota Medan,” *Bilangan : Jurnal Ilmiah Matematika, Kebumian dan Angkasa*, vol. 2, no. 3, pp. 53–60, May 2024, doi: 10.62383/bilangan.v2i3.51.