Mathematical Journal of Modelling and Forecasting
Vol. 3, No. 1, June 2025, pp. 47-55

Risk Comparison in Optimal Portfolios: A Study of Value at Risk (VaR) and Tail Value at Risk (TVaR)

Turnika Afdatul Rafni[1,] and Dina Agustina[2*]

[1,2] Department of Mathematics, Faculty of Mathematics and Natural Sciences, Universitas Negeri Padang, Indonesia
[*] dinagustina@fmipa.unp.ac.id

Abstract. Considering investment risk is something that investors must do before deciding to invest; measuring risk provides an opportunity for investors to get the desired return and minimize losses. This study compares Value at Risk (VaR) and Tail Value at Risk (TVaR) methodologies for measuring portfolio risk. VaR is a commonly used method that provides the maximum loss at a certain confidence level and period. However, VaR is not an effective measure of risk because it does not satisfy one of the axioms of coherent risk measures, i.e., subadditivity. Subsequently, the TVaR measure emerged, which satisfies all the axioms of coherent risk measures, thereby providing a good and effective measure of risk. The optimal portfolio will be formed using the Single Index model, simplifying the Markowitz portfolio model. The Composite Stock Price Index will be the only factor influencing other stocks in this model. The data used data from stocks that were consistently listed on the IDX30 index from 24/10/2022 to 25/10/2024. Based on the result of the analysis of data, the optimal portfolio consists of 5 stocks, i.e., PT Bank Mandiri (BMRI.JK), PT Indofood Sukses Makmur (INDF.JK), PT Bank Central Asia (BBCA.JK), PT Bank Negara Indonesia (BBNI.JK), and PT Barito Pacific (BRPT.JK). Risk measures were compared on the optimal portfolio, using a confidence level of $1 - \alpha = 95\%$, with a daily time period, and an initial investment capital of IDR 1 billion. The estimated VaR risk measure is IDR 15.38 million, while TVaR reaches IDR 23.25 million.

Keywords: Portfolio Optimal, Single Index Model, Value at Risk, Tail Value at Risk.

1 Introduction

Stock investment is one of the most popular forms of investment, because stocks provide an attractive rate of return for an investor [1]. These benefits are certainly followed by risks that investors must understand. Losses can occur because stock prices often experience changes that can occur at any time, both in price decreases and increases [2]. Understanding investment risk is important for an investor if he does not want to experience bankruptcy because he continues to lose money in investment [3], by knowing the measure of risk an investor can determine the amount of funds that must be prepared for investment.[4].

Minimizing risk can be accomplished by diversification, which is spreading funds into several stocks. The stocks chosen are those that provide the highest expected return and certain risks; this combination can be determined by forming an optimal portfolio [5]. The Single Index Model is one of the optimal portfolio formation methods developed by William Sharpe [6], where the purpose of SIM is to simplify the Markowitz model, which is considered quite complex, with the basic idea that stock returns should only be influenced by one factor, namely the Composite Stock Price Index (JCI) [7].

Besides forming an optimal portfolio, it is also important to calculate the potential loss or risk value due to price fluctuations. Risk cannot be completely avoided, but it can be managed and estimated [8]. One commonly used risk measurement method is Value at Risk (VaR). VaR is a quantity to measure the worst expected loss (maximum loss) over a period under normal market conditions at a certain confidence level [9]. VaR can be used to determine the amount of capital needed to cover large losses that may occur [10]. Here are three main methods to calculate VaR, namely the parametric method (covariance-variance method), the Monte Carlo simulation method, and the historical simulation [11].The historical simulation method is a method that overrides the assumption of normally distributed returns and the linear relationship between

portfolio returns and single asset returns [12]. In addition, in the historical simulation, the calculation of VaR is determined by the past value of the resulting stock return [5].

Although it has become a widely used risk measure, VaR still has limitations that do not take into account any losses that exceed the VaR level [13], because in the worst conditions it is possible that the value of losses obtained by investors can exceed the VaR value, and VaR does not always fulfill the axiom of subadditivity [14]. This limitation can be overcome using a more informative, useful, and coherent measure of risk, which is called Tail Value at Risk (TVaR), also known as Expected Shortfall (ES) [15].

TVaR calculates the average loss an investor will incur in the event of a loss that exceeds the VaR confidence level [16]. Tail Value at Risk is a risk measure that fulfills all the coherent properties, so it can be said to be a good and effective risk measure [17], and also a risk estimate that can work on both normally and abnormally distributed data [13]. According to Maruddani et al, the TVaR value is always greater than VaR [18].

Trimono and Maruddani have conducted research that compares VaR and Adjusted-Expected Shortfall (Adj-ES) on EXCL.JK and ICBP.JK stocks in the period 09/01/21 to 09/09/22 with the results that [3] recommend using Adj-ES rather than VaR.

Based on the description above, this study will compare the results of estimating the risk measure between VaR and TVaR on the optimal portfolio to be formed with a method that aims to simplify the Markowitz model, i.e., SIM. The data to be used is the daily closing price of stocks that are consistently part of the IDX30 Index in the research period 24/10/2022 to 25/10/2024, which this period was chosen to get accurate research results because this research will use the historical simulation method for VaR calculations, so the more data used, the better the results will be.

2 Research Methods

2.1 Data Type and Source

This study uses stock closing price data for the period 24/10/2022 to 25/10/2024. The closing prices used are the daily closing prices of stocks that consistently formed part of the IDX30 index during the study period, as well as the daily closing prices of the Indonesian Stock Price Index (IHSG) for the same period, obtained from https://finance.yahoo.com/. The data obtained is available on working days in the stock market, meaning there is no data on holidays and weekends. Then it will use risk-free rate data obtained from Bank Indonesia's Rate through Bank Indonesia's official website https://www.bi.go.id/id/statistik/indikator/bi-rate.aspx.

2.2 Research Procedures

This research will begin by forming an optimal portfolio based on stock returns. Return is the result of the investor's courage to take risks on the chosen investment [19]. In this study, the first stage will begin with the process of forming the optimal portfolio, then proceed with the stage of forming the return portfolio, and the last stage is the estimation of risk measures with VaR and TVaR in more detail. The stages are as follows,

Stage 1: Forming the Optimal Portfolio

1. Calculate the return from the closing price of each consistent stock in the IDX30 index and the JCI using the formula [7],

$$R_{it} = \frac{P_{it} - P_{it-1}}{P_{it-1}} \tag{1}$$

Where,

R_{it} is the actual return of stock-i at time t

P_{it} is the closing price of stock-i at time t

and P_{it-1} is the closing price of stock-i at time $t-1$

2. Calculate the expected return of stock using the formula [7],

$$E(R_i) = \frac{1}{n}\sum_{t=i}^{n}(R_{it}) \tag{2}$$

Where,

$E(R_i)$ is the expected return of stock-i at time t

and n is the amount of the closing price data

3. Calculate the variance of stock using the formula [7],

$$\sigma_i^2 = \frac{\sum_{t=i}^{n}[R_{it} - E(R_i)]^2}{n} \quad (3)$$

Where,

σ_i^2 is the variance of stock-i

4. Forming an efficient portfolio, an efficient portfolio has the criteria of the highest return with a certain risk, or the lowest risk with a certain level of return [20].
5. Estimate the parameters β_i using the formula [7],

$$\beta_i = \frac{\sum_{t=i}^{n}(R_{it} - E(R_i)\,(R_{Mt} - E(R_M))}{n\,\sigma_M^2} \quad (4)$$

Where,

R_M is the actual return of JCI

$E(R_M)$ is the expected return of JCI

β_i is a coefficient that measures the change in R_i due to the change in R_M

σ_M^2 is the variance of JCI

6. Estimate the parameters α_i using the formula [7],

$$\alpha_i = E(R_i) - \beta_i\ E(R_M) \quad (5)$$

Where,

α_i is expected value of an independent stock-i returns relative to JCI returns

7. Test the assumptions of SIM on the residuals of each stock. there are two assumptions for the SIM method [7],
   a. The residual error of the stock-i (e_i) is uncorrelated with the residual error of the stock-j (e_j) for all values of i and j.

$$E(e_i, e_j) = 0 \quad (6)$$

   b. e_i is uncorrelated with return JCI (R_M).

$$E(e_i.[R_M - E(R_M)]) = 0 \quad (7)$$

8. Form an optimal portfolio with the SIM method [7],
   a. Sort the stocks based on the largest ERB value to the smallest ERB value.

$$ERB_i = \frac{E(R_i) - R_{BR}}{\beta_i} \quad (8)$$

With

ERB_i is excess return to beta stock-i

R_{BR} is risk-free stock return that can be obtained from the Bank Indonesia interest rate.

   b. Calculate the value of A_i and B_i values of each stock-i, with

$$A_i = \frac{[E(R_i) - R_{BR}]\,\beta_i}{\sigma_{ei}^2} \quad (9)$$

$$B_i = \frac{\beta_i^2}{\sigma_{ei}^2} \quad (10)$$

   c. Calculating the C_i value

$$C_i = \frac{\sigma_M^2 \sum_{j=1}^{i} A_j}{1 + \sigma_M^2 \sum_{j=1}^{i} B_j} \quad (11)$$

d. Finding the cut-off point value where the cut-off point (C*) is the C_i where ERB value last time is still greater than the C_i.
e. Determine the weight (w_i) for each stock included in the optimal portfolio.

$$w_i = \frac{Z_i}{\sum_{j=1}^{k} Z_j} \tag{12}$$

With

w_i is the weighting for stocks-i

With the value of Z_i:

$$Z_i = \frac{\beta_i}{\sigma_{ei}^2}(ERB_i - C^*) \tag{13}$$

Stage 2: Forming the Return of Portfolio

9. Form a portofolio return use the formula [7],

$$R_P = \sum_{i=1}^{n}(w_i . R_i) \tag{14}$$

10. Sorted the portfolio return by the smallest one.

Stage 3: Estimate the Risk Measure

11. Determining percentiles, the percentile attempts to explain the expected investment loss that an investor might incur at a level of α [12].

$$P_\alpha = \alpha \times n \tag{15}$$

with

P_α is the percentile at α

α as significance level

12. Estimate the VaR value using the historical simulation method at a confidence level of $1 - \alpha_1$ using the formula returns [12].

$$VaR_{1-\alpha} = -V_0 \, P_\alpha \sqrt{t} \tag{16}$$

Where,

V_0 is amount of initial fund value

t is set period

13. Estimate the TVaR value at the $1 - \alpha_1$ confidence levels using the formula [16].

$$TVaR_{1-\alpha}(X) = E[X|X > VaR_{1-\alpha}(X)] \tag{17}$$

In this research, excel software will be used to analyse each research stage.

3 Results and Discussion

In this study, the data used is historical daily closing price data from 23 consistent stocks that were part of the IDX30 from 24/10/2022 to 25/10/2024. The list of consistent stocks is contained in Table 1.

3.1 Forming an Efficient Portfolio

There are 483 daily closing price data points for each stock in Table 1. The data is used to calculate the expected return of each stock using equation (2) and the risk using equation (3). Then, the stocks are sorted based on the largest expected return to be analysed and whether or not they are included as efficient portfolios. The sorted stocks are shown in Table 2.

Table 1. Consistent Stock List

No.	Stock Code	Company Name
1.	ADRO.JK	PT Alamtri Resources Indonesia Tbk
2.	ANTM.JK	PT Aneka Tambang Tbk
3.	ARTO.JK	PT Bank Jago Tbk
4.	ASII.JK	PT Astra International Tbk
5.	BBCA.JK	PT Bank Central Asia Tbk
6.	BBNI.JK	PT Bank Negara Indonesia (Persero) Tbk
7.	BBRI.JK	PT Bank Rakyat Indonesia (Persero) Tbk
8.	BMRI.JK	PT Bank Mandiri (Persero) Tbk
9.	BRPT.JK	PT Barito Pacific Tbk
10.	BUKA.JK	PT Bukalapak.com Tbk.
11.	CPIN.JK	PT Charoen Pokphand Indonesia Tbk
12.	GOTO.JK	PT GoTo Gojek Tokopedia Tbk
13.	INCO.JK	PT Vale Indonesia Tbk
14.	INDF.JK	PT Indofood Sukses Makmur Tbk
15.	ITMG.JK	PT Indo Tambangraya Megah Tbk
16.	KLBF.JK	PT Kalbe Farma Tbk
17.	MDKA.JK	PT Merdeka Copper Gold Tbk
18.	PGAS.JK	PT Perusahaan Gas Negara Tbk
19.	PTBA.JK	PT Bukit Asam Tbk
20.	SMGR.JK	PT Semen Indonesia (Persero) Tbk
21.	TLKM.JK	PT Telekomunikasi Indonesia Tbk
22.	UNTR.JK	PT United Tractors Tbk
23.	UNVR.JK	PT Unilever Indonesia Tbk

Table 2. Sorted Expected Return and Variance

Stock Code	Expected Return	Variance
BRPT.JK	0.00121	0.00150
BMRI.JK	0.00080	0.00027
BBNI.JK	0.00051	0.00024
BBCA.JK	0.00047	0.00015
INDF.JK	0.00044	0.00015
BBRI.JK	0.00024	0.00025
ADRO.JK	0.00010	0.00048
CPIN.JK	0.00004	0.00035
ANTM.JK	-0.00006	0.00039
UNTR.JK	-0.00022	0.00035
KLBF.JK	-0.00027	0.00033
PTBA.JK	-0.00032	0.00047
PGAS.JK	-0.00035	0.00033
ASII.JK	-0.00037	0.00025
ARTO.JK	-0.00044	0.00190
MDKA.JK	-0.00051	0.00075
TLKM.JK	-0.00074	0.00023
INCO.JK	-0.00077	0.00046
BUKA.JK	-0.00092	0.00090
SMGR.JK	-0.00094	0.00038
ITMG.JK	-0.00095	0.00038
GOTO.JK	-0.00117	0.00186
UNVR.JK	-0.00172	0.00041

Based on Table 2, the stocks included in the efficient portfolio in this study are based on stocks with the highest expected return for a given level of risk. Based on this, five stocks were selected, including BRPT.JK, BMRI.JK, BBNI.JK, BBCA.JK, and INDF.JK, as shown in Table 2. The reason why stocks are below

INDF.JK was not included in the efficient portfolio because it offers a lower expected return than INDF.JK but carries a higher risk than INDF.

3.2 Forming an Optimal Portfolio with Single Index Model

Model Assumption Test

Based on the correlation matrix results for the two assumptions in SIM, the five stocks in the efficient portfolio meet all assumptions.

Selection of Stocks Included in the Optimal Portfolio

The selection of optimal portfolio candidates is carried out by comparing the ERB_i value obtained by equation (8) with the cut point value (C_i) obtained based on equation (11). First, the ERB results for each stock are sorted from the largest to the smallest.

Stocks that enter the optimal portfolio provide ERB values greater than the cut point value. The comparison results are shown in Table 3.

Table 3. Comparison of ERB Value with C_i

Stock Code	ERB_i	C_i
INDF.JK	0.00090	0.00003
BRPT.JK	0.00048	0.00010
BMRI.JK	0.00046	0.00021
BBCA.JK	0.00034	0.00024
BBNI.JK	0.00033	0.00025

Based on Table 3, when considering the ERB value of INDF.JK stock to the ERB of BBNI.JK stock, they are all greater than the C_i value obtained, therefore the C_i value taken as the cut-off point is the largest C_i The cut-off point value obtained is 0.00025. When compared again, the ERB value of other stocks is also still greater than the cut-off point value set, so it is concluded that the five stocks above are included in the optimal portfolio.

Calculate the Weight of Each Stock in the Portfolio

Equation (12) is used to obtain each stock's weight; the results are shown in Table 4.

Table 4. Weight for Each Stock

Stock Code	Weight	Weight (%)
INDF.JK	0.30336	30%
BRPT.JK	0.08276	8%
BMRI.JK	0.34778	35%
BBCA.JK	0.16624	17%
BBNI.JK	0.09985	10%

Table 4 shows that BMRI. JK has the highest investment weight, at 35%.

3.3 Calculating Portfolio Return

Because the historical simulation method will be used for risk measurement in this study, the daily portfolio return will first be formed by summing up all the results of the multiplication between the actual daily return and the weight of the stock in the optimal portfolio, as shown in formula (14). The results are obtained in Table 5. More complete stock portfolio returns for October 24, 2022, to October 25, 2024, can be seen at https://tinyurl.com/ReturnPortofolio.

3.4 Calculating Value at Risk

In this study, VaR is calculated using the historical simulation method using equation (16), with a confidence level $1 - \alpha = 95\%$ the first step taken is to determine the location of the percentile through the portfolio return in Table 6, which must be sorted from smallest to largest, based on equation (15), then with $\alpha = 5\%$ and the amount of data is 482, the location of the percentile is:

$$P_{0,05} = 0.05 \times 482 \approx 24$$

Table 5. Portofolio Return

Period	Portofolio Return
25/10/2022	-0.00757
26/10/2022	-0.00073
27/10/2022	0.00718
28/10/2022	0.00332
31/10/2022	0.01626
⋮	⋮
21/10/2024	-0.00182
22/10/2024	-0.00485
23/10/2024	0.00548
24/10/2024	-0.00560
25/10/2024	0.00314

From the results obtained, the location of the percentile at the 95% confidence level is in the 24th order of the sorted portfolio returns. So that the value of $P_{0.05} = 0.01538$. Suppose the initial capital in investment is Rp1,000,000,000, with a daily period, then the estimated VaR value for the optimal portfolio using equation (16) is as follows:

$$VaR_{0.95} = -1{,}000{,}000{,}000 \times 0.01538_{\alpha} \times \sqrt{1}$$

$$VaR_{0.95} = 15{,}379{,}498$$

Based on the above calculation, it can be concluded that investing in a portfolio with an initial capital of Rp1,000,000,000, there is 95% confidence that the loss will not exceed Rp15,379,498 within one day after October 25, 2024.

3.5 Calculating Tail Value at Risk

TVaR is calculated using equation (17) so that the TVaR value at the $1 - \alpha = 95\%$ confidence level and initial capital in investment is Rp1,000,000,000 is obtained as follows:

Table 6. Estimated Value of TVaR

	$TVaR_{0,95}$
Portofolio	Rp23,253,190

Based on the calculation results shown in Table 6, it can be concluded that the amount of loss that investors will bear if the worst condition occurs, namely the loss value exceeds the VaR estimate with a 95% confidence level, is Rp23,253,190.

From the two results of the measure, it can be seen that TVaR will always provide a measure of risk greater than VaR, which is certainly because TVaR provides information to investors regarding the estimated losses that will be borne in the event of the worst conditions. In contrast, VaR provides information that under normal market conditions, the losses an investor bears will not exceed a certain point with a certain level of confidence and a fixed period of time. Both risk measures can be used as a benchmark for investors in providing funds in the event of the worst conditions in the investment period.

4 Conclusion

Based on the analysis results, investors are advised to invest in 5 stocks out of 23 that are consistent in the IDX30 index. Each stock has a different investment fund weight. In this study, the following were found: Indofood Sukses Makmur Tbk (INDF.JK) with an investment weight of 30%, Barito Pacific Tbk (BRPT.JK) with an investment weight of 8%, Bank Mandiri Tbk (BMRI.JK) with an investment weight of 35%, and Bank Central Asia Tbk. (BBCA.JK) with an investment weight of 17%, Bank Negara Indonesia Tbk (BBNI.JK) with an investment weight of 10%. Investors are advised to allocate the largest investment fund, 35%, to the BMRI.JK stock.

By following the above investment pattern, investors are assured that with an initial investment of IDR 1 billion, under normal market conditions, with a 95% confidence level as of October 26, 2024, the maximum loss experienced by investors will not exceed IDR 15,379,498. However, in the worst-case scenario, the average loss experienced by investors will be IDR 23,253,190. Investors can choose which risk measure to use based on their preferences; however, it is evident that the risk measure generated by TVaR is larger than VaR. Therefore, if investors follow the TVaR risk measure, they must prepare more funds to anticipate losses.

References

[1] Bursa Efek Indonesia, "Saham." Accessed: Sep. 26, 2024. [Online]. Available: https://www.idx.co.id/id/produk/saham/

[2] D. Adrianto, D. A. I. Maruddani, and Tarno, "Pengukuran Risiko Glue-Value-at-Risk Pada Data Distribusi Elliptical," *Jurnal Gaussian* , vol. 9, no. 1, pp. 75–86, 2020.

[3] T. Trimono and D. A. Maruddani, "Comparison Between Value at Risk and Adjusted Expected Shortfall: A Numerical Analysis," *Barekeng: Jurnal Ilmu Matematika dan Terapan*, vol. 17, no. 3, pp. 1347–1358, Sep. 2023, doi: 10.30598/barekengvol17iss3pp1347-1358.

[4] A. Dimas Adrianto, M. Azhari, and Khairunnisa, "Analisis Value At Risk Dengan Metode Historis, dan Monte Carlo Dalam Saham Sub Sektor Rokok (Studi Kasus Pada Saham Gudang Garam Dan Hm Sampoerna).," *e-Proceeding of Management*, vol. 4, no. 2, 2017.

[5] A. Solihatun, L. Gubu, Aswani, E. Cahyono, and L. Ode Saidi, "Perhitungan Value at Risk (VaR) pada Portofolio Saham IDX Sektor Keuangan (IDXFinance) Menggunakan Metode Simulasi Historis (Historical Simulation Method)," *Jurnal Matematika, Komputasi dan Statistika*, vol. 3, no. 1, 2023.

[6] E. Tandelilin, *Pasar Modal Manajemen Portofolio & Investasi*. Daerah Istimewa Yogyakarta: Penerbit PT Kanisius, 2017.

[7] J. M. Prof. , Dr. , MBA. , Ak. , CMA. , CA. Hartono, *Teori dan Praktik Portofolio dengan Excel* . Jln. Raya Lenteng Agung No. 101 Jagakarsa Jakarta Selatan : Penerbit Salemba Empat, 2013.

[8] Mustafian, Mauliddin, and A. M. Abdal, "Penerapan Value-at-Risk dan Conditional-Value at-Risk dalam Pengukuran Risiko Portofolio Optimal Menggunakan Pendekatan Simulasi Monte Carlo," *Jurnal Riset dan Aplikasi Matematika*, vol. 08, no. 01, pp. 39–50, 2024.

[9] P. Jorion, *Value at Risk: The New Benchmark for Managing Financial Risk*, 2nd ed. USA: McGraw-Hill, 2001.

[10] R. Ruhiyat, B. Setiawaty, and M. Z. Dhuha, "Value at Risk dan Tail Value at Risk dari Peubah Acak Besarnya Kerugian yang Menyebar Alpha Power Pareto," *Jambura Journal of Mathematics*, vol. 5, no. 1, pp. 67–82, Jan. 2023, doi: 10.34312/jjom.v5i1.16586.

[11] R. Rahmawati, A. Rusgiyono, A. Hoyyi, and D. A. I. Maruddani, "Expected Shortfall Untuk Mengukur Risiko Kerugian Petani Jagung," *MEDIA STATISTIKA*, vol. 12, no. 1, p. 117, Jul. 2019.

[12] T. S. Nuryanto, A. Prahutama, and A. Hoyyi, "Historical Simulation Untuk Menghitung Value At Risk Pada Portofolio Optimal Berdasarkan Single Index Model Menggunakan Gui Matlab (Studi Kasus: Kelompok Saham JII Periode Juni-November 2017)," *Jurnal Gaussian*, vol. 7, no. 4, pp. 408–418, 2018.

[13] N. F. Rizani, Mustafid, and Suparti, "Penerapan Metode Expected Shortfall Pada Pengukuran Risiko Investasi Saham Dengan Volatilitas Model Garch," *Jurnal Gaussian*, vol. 8, no. 2, pp. 184–193, 2019.

[14] I. B. A. Darmayuda, K. Dharmawan, and K. Sari, "Estimasi Expected Shortfall dalam Optimalisasi Portofolio dengan Metode Downside Deviation pada Saham IDXHEALTH," *E-Jurnal Matematika*, vol. 12, no. 2, p. 114, May 2023, doi: 10.24843/mtk.2023.v12.i02.p408.

[15] P. Artzner *et al.*, "Coherent Measures of Risk," 1998.

[16] S. A. Klugman, H. H. Panjer, and G. E. Willmot, *Loss Models From Data to Decisions*, 5th ed. USA: John Wiley and Sons, Inc,, 2019.

[17] E. K. Dewi, D. Ispriyanti, and A. Rusgiyono, "Expected Shortfall Pada Portofolio Optimal Dengan Metode Single Index Model (Studi Kasus pada Saham IDX30)," *Jurnal Gaussian*, vol. 10, no. 2, pp. 269–278, 2021.

[18] D. A. I. Maruddani and T. D. Astuti, "Risiko Investasi Saham Second Liner Dengan Tail Value at Risk," *Jurnal Ilmiah Manajemen*, vol. 11, no. 2, 2021.

[19] A. Gunawan, R. Fajriyah, M. A. Bimakasa, and S. N. Untari, "Analysis of Portfolio Formation on the LQ45 Stocks Index, Using the Markowitz and Single Index Models," *Barekeng*, vol. 18, no. 4, pp. 2363–2374, Dec. 2024, doi: 10.30598/barekengvol18iss4pp2363-2374.

[20] V. R. A. Priyantono, D. A. I. Maruddani, and I. T. Utami, "Analisis Portofolio Optimal Menggunakan Model Indeks Tunggal dan Pengukuran Value at Risk dengan Simulasi Monte Carlo (Studi Kasus: Exchange Traded Fund di Bursa Efek Indonesia Periode Januari 2021 – Juni 2022)," *Jurnal Gaussian*, vol. 12, no. 2, pp. 158–165, Jul. 2023, doi: 10.14710/j.gauss.12.2.158-165.