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Enhancing Oil Field Investment Decisions Using Spiral Dynamics Optimization

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Abstract. Investment is an activity in the present to obtain profits in the future. One method of investment assessment is calculating Net Present Value (NPV) using Discounted Cash Flow (DCF). To determine the right time to carry out an investment in order to obtain maximum profits, it is necessary to determine a time limit for when the investment should be carried out or not. Then, the threshold curve (optimal implementation time limit) for the investment will first be determined. After obtaining the threshold curve for each investment option, the NPV value for each investment option will then be determined. The best investment choice is the one that provides the highest NPV value. The metaheuristic method is an effective optimization method for determining the optimal investment implementation time limit (threshold curve). One metaheuristic method for determining threshold curves is the Spiral Dynamics Optimization algorithm developed by Kenichi Tamura and Keiichiro Yasuda, a search algorithm inspired by phenomena in nature such as water speed, air pressure speed, Nautilus Shells, and spiral galaxy shapes. The results of this research are areas that are above the threshold curve, which can implement the project, while areas that are below the curve are not recommended for implementing the project.

Keywords: Investment, Optimal, Spiral Dynamics Optimization, Threshold Curve

1 Introduction

In general, investment appraisal is carried out to assess the benefits and costs of an investment [1]. In investment appraisal, the estimated costs and benefits that will be obtained from an investment are analyzed, and the results are used to determine whether the investment is worth making or not. If the benefits of an investment exceed its costs at the level expected by the decision maker, then the investment will be executed, and if not, then vice versa. Determining investment decisions in an oil field requires a fairly in-depth analysis of several uncertainty factors. There are two uncertainty factors: technical uncertainty and market uncertainty [2]. In this case, only market uncertainty, namely the price of oil, is considered.

Analysis of market uncertainty will help investment managers in making investment decisions in oil fields, namely the decision to immediately make the investment or wait for better market conditions [3]. When there are many investment options to choose from, in this case, where there are several investment alternatives, selecting the best alternative is a very important and complex decision because it is a large investment opportunity. Several parameters will be considered. This investment analysis aims to obtain optimal investment decisions for an oil reservoir by considering market uncertainty, especially oil prices.

2 Theoretical Basis

2.1 Two-Dimensional Spiral Dynamics Optimization Algorithm

To form a spiral, a rotation matrix with a rotation angle of θ is required. The following is a 2-dimensional spiral optimization algorithm for global maximization problems [4].

Input:

m ($m \geq 2$): number of search starting points

k_{max} ($k_{max} > 0$): maximum number of iterations
θ ($0 \leq \theta \leq 2\pi$: rotation angle
r ($0 < r < 1$): convergence rate
$f(x)$: objective function

Process:

1. Step 0: Preparation

   Determine the number of search points $m \geq 2$ with parameters $0 \leq \theta \leq 2\pi$ and $0 < r < 1$ from $S_2(r, \theta)$ and the maximum number of iterationsk_{max} . Then set the value $k = 0$.
2. Step 1: Initialization

   Determine the initial points $x_i \in \mathbb{R}^2$ for $i = 1,2, \dots, m$ in the feasible region and the center point $x^* = x_{i^*}$ with $i^* = \arg\max f(x_i(0))$ with $i = 1,2, \dots, m$.
3. Step 2: Update x_i
   For $i = 1,2, \dots, m$,
   then $x_i(k+1) = S_2(r, \theta)x_i(k) - (S_2(r, \theta) - I_2)x^*$
4. Step 3: Update x^*
   Determine the new center point $x^* = x_{i^*}(k+1)$ with
   $i^* = \arg\max f(x_i(k+1))$ for $i = 1,2, \dots, m$.
5. Step 4: Test the stopping criteria
   If $k = k_{max}$ then the process is complete. Otherwise, set $k = k + 1$ and return to Step 2.

Output:

Maximum value and global maximum solution of the function $f(x)$.

2.2 n-Dimensional Spiral Dynamics Optimization Algorithm

The following presents an n-dimensional spiral optimization algorithm for global maximization problems. There are no fundamental differences between 2-dimensional spiral optimization algorithms, except for the process of determining the starting point and the rotation matrix used [4].

Input:

m ($m \geq 2$) : number of search starting points

k_{max} ($k_{max} > 0$: maximum number of iterations
θ ($0 \leq \theta \leq 2\pi$) : rotation angle
r ($0 < r < 1$) : convergence rate
$f(x)$: objective function

Process:

1. Step 0: Preparation

   Determine the number of search points $m \geq 2$ with parameters $0 \leq \theta \leq 2\pi$ and $0 < r < 1$ from $S_n(r, \theta)$ and the maximum number of iterations k_{max} . Then set the value $k = 0$.
2. Step 1: Initialization

   Determine the initial points $x_i \in \mathbb{R}^2$ for $i = 1,2, \dots, m$ in the feasible region and the center point $x^* = x_{i^*}$ with $i^* = \arg\max f(x_i(0))$ with $i = 1,2, \dots, m$.
3. Step 2: Update x_i
   For $i = 1,2, \dots, m$,
   then $x_i(k+1) = S_n(r, \theta)x_i(k) - (S_n(r, \theta) - I_2)x^*$
4. Step 3: Update x^*
   Determine the new center point $x^* = x_{i^*}(k+1)$ with
   $i^* = \arg\max f(x_i(k+1))$ for $i = 1,2, \dots, m$.
5. Step 4: Test the stopping criteria
   If $k = k_{max}$ then the process is complete. Otherwise, set $k = k + 1$ and return to Step 2.

Output:

Maximum value and global maximum solution of the function $f(x)$.

3 Results and Discussion

3.1 Investment Analysis to Determine Optimal Investment

Analysis of market uncertainty will help investment managers in making investment decisions in oil fields, namely the decision to make the investment or wait for better market conditions immediately. When there are many investment options to choose from, selecting the best alternative is a very important and complex decision because it is a large investment opportunity, and there are several parameters to be considered [5].

Each alternative provides an optimal exercise curve, or what is called a threshold curve, which determines the critical value for the optimal exercise area [6]. All threshold curves describe decision rules that maximize the value of the alternatives.

Basically, when calculating each investment alternative in oil fields, the first thing to pay attention to is the maximum NPV value; in other words, the best alternative is the alternative that has the largest NPV value. The NPV equation for this case is [7]:

$$\mathrm{NPV} = \frac{\sum_{t=1}^{N} qS_t B}{N} - D \tag{1}$$

It can be noted that the NPV formula can be used when all parameters are known. However, in reality, all these parameters can change over time and are a source of uncertainty. Therefore, in this case, it is considered that there is only market uncertainty (only oil price uncertainty is considered).

3.2 Threshold Curve and Its Constraints

The threshold curve can be approximated using the natural logarithm function, namely $a + b\ln(\tau)$ and adding a free point which is located near the option's maturity date, or which occurs in the time period τ (the option's maturity time) [8]. This logarithmic function is used because it is the best approximation for determining the threshold curve obtained from the finite difference method [9]. Waiting area (not exercising) between the areas formed by the threshold curve of the alternative. This waiting area is also approximated using the logarithm function, namely $a_W - b_W \ln(\tau)$ and a free point. The coefficient values of the logarithm function (a, b, a_W, b_W) and free points will be determined using the Spiral Dynamics Optimization algorithm [10].

In defining domain constraints, we need to calculate the critical oil price at maturity. This is done because an option will be valuable if it is exercised at maturity, where the NPV value of the option must be greater than or equal to zero or equal to the NPV of the investment alternative with the lowest investment costs.
The linear constraints for each threshold curve and waiting curve are as follows

$$\begin{gathered} a + b\ln(0.1 + dt) \geq FreePoint \\ a_W - b_W \ln(0.1 + dt) \leq FreePoint \\ a + b\ln(0.1 + dt) \geq 0 \\ a_W - b_W \ln(0.1 + dt) \geq 0 \end{gathered} \tag{2}$$

Free points for each alternative are selected for the same time. The logarithmic function curve starts at time (0.1+Δt) where Δt is the time interval [10].

The threshold curve describes the decision rule for developing an oil field. By simulating the oil price $P(t)$. In this case, the threshold curve will be determined using the Spiral Dynamics Optimization Algorithm.

3.3 Threshold Curve for Each Investment Option Using the Spiral Dynamics Optimization Algorithm

In this section, the coefficient values of the logarithmic function (a, b, a_W, b_W) and free points will be determined using the Spiral Dynamics Optimization algorithm, which meets domain constraints and linear constraints on each threshold curve [11]. So, the Spiral Dynamics Optimization algorithm used is the three-dimensional Spiral Dynamics Optimization algorithm because in one threshold curve function, there are three function coefficient values to be determined, namely a, b and *free point* [11]. However, in general, for each curve, the threshold can be determined in the same way.

The following is a three-dimensional Spiral Dynamics Optimization algorithm that satisfies domain constraints and linear constraints. Because what will be searched for are three values that will maximize the function, namely a_1, b_1, $Free\ Pt_1$, a three-dimensional spiral optimization algorithm will be used for global maximization problems [12][13].

Input:

$m = 1000$	: number of search starting points
$k_{max} = 500$	: maximum number of iterations
$\theta = \pi/4$	: rotation angle
$r = 0{,}95$	: convergence rate
$f_1 = a_1 + b_1 ln(0.1 + dt) - FreePoint_1$	: objective function

Process:

1. Step 0: Preparation

Determine the number of search points $m = 1000$ with parameter $\theta = \pi/4,\ r = 0.95$ from $S_3(r, \theta)$ and the maximum number of iterations $k_{max} = 500$. Then set the value $k = 0$.

2. Step 1: Initialization

Determine the initial points $x_i \in \mathbb{R}^2 a_{1_i}, b_{1_i}, FreePoint_{1_i} \in \mathbb{R}^3$ for $i = 1,2,\ldots,1000$

in areas that meet the threshold curve domain constraints with value $a_1, b_1, FreePoint_1$ is a random number and a central point $a_1{}^* = a_{1_{i^*}}$, $b_1{}^* = b_{1_{i^*}}$, $FreePoint_1{}^* = FreePoint_{1_{i^*}}$ with

$i^* = \arg\max f((a_{1_i}, b_{1_i}, FreePoint_{1_i})(0)),\ i = 1,2,\ldots,1000.$

3. Step 2: Update x_i

For $i = 1,2,\ldots,1000$, then

$$\begin{bmatrix} a_{1_i}(k+1) \\ b_{1_i}(k+1) \\ FreePt_{1_i}(k+1) \end{bmatrix} = S_3(r,\theta) \begin{bmatrix} a_{1_i}(k) \\ b_{1_i}(k) \\ FreePt_{1_i}(k) \end{bmatrix} - (S_3(r,\theta) - I_3) \begin{bmatrix} a_1{}^* \\ b_1{}^* \\ FreePt_1{}^* \end{bmatrix}$$

With matrix S as follows [14],

$$S_3(r,\theta) = rR^{(3)}(\theta) \begin{bmatrix} a_{1_i}(k) \\ b_{1_i}(k) \\ FreePoint_{1_i}(k) \end{bmatrix}$$

$$S_3(r,\theta) = rR_{1,2}{}^{(3)}(\theta) \times R_{1,3}{}^{(3)}(\theta) \times R_{2,3}{}^{(3)}(\theta) \times \begin{bmatrix} a_{1_i}(k) \\ b_{1_i}(k) \\ FreePoint_{1_i}(k) \end{bmatrix}$$

$$S_3\left(0.95, \frac{\pi}{4}\right) = 0.95 \begin{bmatrix} \cos\frac{\pi}{4} & -\sin\frac{\pi}{4} & 0 \\ \sin\frac{\pi}{4} & \cos\frac{\pi}{4} & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} \cos\frac{\pi}{4} & 0 & -\sin\frac{\pi}{4} \\ 0 & 1 & 0 \\ \sin\frac{\pi}{4} & 0 & \cos\frac{\pi}{4} \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\frac{\pi}{4} & -\sin\frac{\pi}{4} \\ 0 & \sin\frac{\pi}{4} & \cos\frac{\pi}{4} \end{bmatrix} \times \begin{bmatrix} a_{1_i}(k) \\ b_{1_i}(k) \\ FreePoint_{1_i}(k) \end{bmatrix}$$

$$S_3(0.95, \pi/4) = \begin{bmatrix} -0.4750 & -0.8109 & 0.1391 \\ 0.4750 & 0.1391 & -0.8109 \\ 0.6718 & 0.4750 & 0.4750 \end{bmatrix} \begin{bmatrix} a_{1_i}(k) \\ b_{1_i}(k) \\ FreePoint_{1_i}(k) \end{bmatrix}$$

4. Step 3: Update x^*

Determine the new center point

$$\begin{bmatrix} {a_1}^* \\ {b_1}^* \\ {FreePoint_1}^* \end{bmatrix} = \begin{bmatrix} a_{1_{i^*}}(k+1) \\ b_{1_{i^*}}(k+1) \\ FreePoint_{1_{i^*}}(k+1) \end{bmatrix}$$

with $i^* = argmaxf((a_{1_i}, b_{1_i}, FreePoint_{1_i})(k+1))$, $i = 1,2,\dots,1000$.

5. Step 4: Testing stopping criteria

If $k = k_{max} = 500$ then the process is complete. If not, then set $k = k + 1$ and return to Step 2.

Output:

Because the values $a_1, b_1, FreePoint_1$ are random numbers, to get optimal results, run the program repeatedly. The results of the running program can be seen in Table 1.

Table 1. Results of the Running Program of Threshold Curve

Iteration	$\boldsymbol{a_1}$	$\boldsymbol{b_1}$	$\boldsymbol{FreePt_1}$
1	16.5803	0.1023	13.3781
2	18.0327	0.403	14.0774
3	17.3921	1.7917	12.5278
4	16.4666	0.4167	14.42
5	18.1811	0.8866	13.1851
6	14.4961	0.1568	12.8915
7	17.1289	0.1489	12.8063
8	18.0985	0.4526	13.9018
9	18.0748	1.2999	13.2628
10	18.1614	0.6416	13.4819
11	17.3946	1.2163	14.4634
12	18.043	1.1054	15.1588
13	18.7342	1.027	12.646
14	16.7299	0.5925	15.1375
15	15.765	1.488	12.5359
16	18.2462	0.2146	13.5066
17	15.7928	0.2885	13.8071
18	14.7723	0.185	13.5467
19	16.4417	0.5622	15.0875
20	18.4196	0.4504	12.9305

After carrying out 20 runs with Matlab, optimal results were obtained in the 15th run with the following results:

${a_1}^* = 15.7650$, ${b_1}^* = 1.4880$, ${FreePoint_1}^* = 12.5359$

After obtaining the value of each coefficient of the logarithmic function of the threshold curve, the threshold curve can be drawn. Threshold curve logarithmic function:

$$15.7650 + 1.4880\ln(0.1 + dt)$$

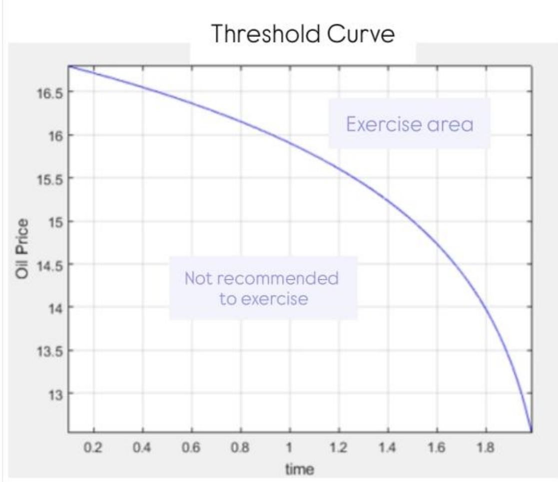

Fig 1. Threshold Curve

Areas above the threshold curve are areas that can be exercised (i.e., the project can be implemented). In contrast, areas below the curve are areas that are not recommended for exercise (i.e., not implemented the project) [15].

Objective function of the waiting area threshold curve:

$$f_4 = a_4 - b_4 \ln(0.1 + dt) - FreePoint_4$$

With domain constraints as follows:

$$12.5 \le a_4 \le 18.75$$
$$0 \le b_4 \le \frac{60-18.75}{\ln\,(2+\Delta t)}$$
$$12.5 \le Free\ Pt_4 \le 18.75$$

The logarithmic function of the threshold curve of the waiting area is

$$18.1553 - 2.9850 \ln(0.1 + dt)$$

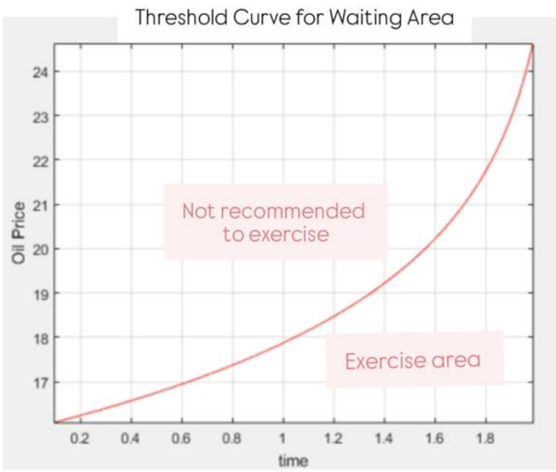

Fig 2. The Threshold Curve for the Waiting Area

Where in the waiting area threshold curve, the area below the threshold curve is the area where options can be exercised, while the area above the curve is the area where it is recommended not to exercise options (waiting area) [15].

4 Conclusion

In implementing an investment, an investor needs to determine when the right time is to carry out a project in order to obtain maximum profits. It is necessary to determine the boundary between whether the project should be implemented or not. Then, the threshold curve (optimal implementation time limit) for the investment will first be determined. The Spiral Dynamics Optimization (SDO) algorithm, developed by Kenichi Tamura and Keiichiro Yasuda, is a metaheuristic method inspired by the logarithmic spiral phenomenon in nature. It can be used to determine the optimal coefficient value of the logarithmic function of the threshold curve. After obtaining the threshold curve for each investment option, the NPV value for each investment option will be determined. The best investment choice is the one that provides the highest NPV value. The results of this research show that areas above the threshold curve can implement the project, while areas below the curve are not recommended.

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