Mathematical Journal of Modelling and Forecasting
Vol. 2, No. 2, December 2024, pp. 46-57

# Variance and Semi-Variance with a Multi-Objective Approach Using the Spiral Optimization Method

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**Abstract.** In minimizing the risk faced by investors while maximizing returns, it is necessary to study different risk measures for portfolio optimization, namely mean-variance, and semi-variance, so that it can provide a deeper understanding of how each approach works in various market conditions. The mean-variance approach measures risk based on the total variance of portfolio returns. While the semi-variance approach only focuses on downside risk, which is the risk of loss that is more relevant to investors who tend to be conservative. By comparing these two risk measures, investors can understand the trade-offs in choosing a portfolio management strategy. To conduct a study on portfolio optimization, the author uses a multi-objective optimization approach on the mean-variance and semi-variance models, which will be solved with a spiral model. The results of this study are that the spiral model with a simple case that does not involve high dimensions can be solved quickly. However, for high dimensions with significant maximum spread points and iterations, the algorithm in this Matlab programming runs slowly, so it is ineffective in computation. This spiral method is suspected of having several solutions trapped in local minima, or the results obtained have not converged, so the resulting Pareto front is not optimal.

**Keywords:** Portfolio optimization, mean-variance, semi-variance, and spiral.

## 1 Introduction

In the world of investment, the main goal of an investor is to maximize the return or profit obtained from his investment while minimizing the risks faced [1]. However, high returns are generally accompanied by high risks [2], so investors must find an effective strategy to balance risk and profit. For example, organizations often face a dilemma between efficiency and profitability in the business world. In manufacturing, companies want to maximize profits through high production and sales and must consider operational cost efficiency and resource constraints. This tension makes decision-making complex because the priority of one goal often comes at the expense of another.

For example, a factory with limited capacity may want to increase profits by producing more goods, but overproduction can increase energy, raw materials, and labour costs. On the other hand, focusing too much on cost reduction can reduce the capacity to meet market demand, thus reducing revenue. Situations like this require an approach that can identify the optimal solution based on the preferences between two or more competing objectives. One way can be done is by combining assets (shares) into a portfolio to form an optimal portfolio according to an investor's goals. Suppose an investor wants to minimize risk with a desired target return or maximize return with a target risk. This problem is known as a single-objective portfolio problem involving one objective. It would be better if return and risk could be optimized simultaneously. This problem is known as a multi-objective portfolio optimization problem.

Mean-Variance was first developed by Markowitz in 1952. The portfolio is formed with the average and standard deviation of stock returns based on the relationship between the stocks that form the portfolio [3]. This portfolio is then known as the Markowitz model portfolio. This portfolio forms an efficient portfolio that offers risk with a certain level of return. From several efficient portfolios formed through this model, an optimal portfolio can be selected with the smallest standard deviation that measures the risk in the portfolio.

Several approaches can be used to obtain this efficient portfolio. Mean-variance assumes that stock returns are normally distributed. In reality, not all stock returns are normally distributed. That is why another risk measure is used for Multi-Objective problems, namely semi-variance, where semi-variance does not require any distribution.

Chang conducted a study on the problem of multi-objective portfolio optimization with the Mean-Variance and Semi-Variance models using the Genetic Algorithm (GA) method with the help of C++ software. This study showed that the GA method effectively solves portfolio optimization problems in several risk sizes. The objectives of this article are as follows: First, to solve the portfolio optimization problem involving multi-objective problems for the mean-variance model using the spiral optimization method. Second, the spiral optimization method was used to solve the semi-variance model's portfolio optimization problem involving multi-objective problems [2].

This article discusses a multi-objective case for mean-variance and semi-variance models using spirals, where this approach is used to optimize investment portfolios by considering risk and return. The spiral approach is used to explore Pareto-optimal solutions, which include various alternatives that are balanced between risk minimization (variance or semi-variance) and return maximization (mean). The analysis is carried out by simulation of field data. From the background above, it is necessary to make a study related to portfolio optimization for multi-objective cases for mean-variance and semi-variance models using spirals. This study aims to provide a more comprehensive solution to the conflict between the objectives of maximizing returns and minimizing risk.

The spiral approach is expected to obtain an optimal solution that efficiently balances both objectives, especially in a portfolio with various assets. This research will also cover the application of the spiral method in producing Pareto-optimal solutions. The results of this study are expected to contribute significantly to the development of portfolio optimization methods and be a reference for investors in making more appropriate investment decisions based on their risk and return preferences.

## 2 Research Methods

To conduct a study on portfolio optimization, the author needs first to explain optimization involving multi-objective problems in the mean-variance and semi-variance models. Multi-objective optimization is an approach that aims to solve problems with more than one objective function, where these functions often conflict. In the mean-variance model, the main objective is to maximize the portfolio's average return (mean) while minimizing variance as a measure of risk. Meanwhile, the semi-variance model focuses on minimizing downside risk, a loss that occurs below a certain level, which is considered more relevant for most investors [4-5].

The author will also review the basic concept of the Pareto front, which is a collection of optimal solutions in multi-objective optimization. The Pareto front solutions offer various trade-off options between return and risk, allowing investors to choose the portfolio that best suits their preferences and risk tolerance. In addition, the spiral method will be explained as a technique for finding optimal solutions in a multi-objective parameter space. Emphasis will be given on how this method works to find Pareto-optimal solutions with high efficiency, then explain the spiral method and conduct a trial application of the spiral method on portfolio optimization for multi-objective problems (mean-variance and semi-variance models) [6].

## 3 Theoretical Basis

### 3.1 Multi-Objective Portfolio Optimization

An investor wants to maximize returns and minimize risks simultaneously, so finding the optimal solution is necessary. In this sub-chapter, we will discuss multi-objective portfolio optimization if both objectives are optimized simultaneously with several objective functions.

For minimization problems, in general, it can be written:

$$\text{Minimize } f_l(x), l = 1,2, \dots, L \tag{1}$$

Constraint $g_j(x) \leq 0, j = 1,2, \dots, J$ (2)
$h_k(x) \leq 0, k = 1,2, \dots, K$ (3)
$x_i^{min} \leq x_i \leq x_i^{max}, i = 1,2, \dots, n$ (4)

where: $f_l(x)$ = The function to be minimized.
$g_j(x), h_k(x)$= Constraint function.

In the optimization problem above, there are L objective functions where each objective function will be minimized or maximized. The maximizing problem can be converted into a minimizing problem by multiplying the objective function by -1. so now the goal of each objective function is the same, namely to minimize.

A solution that satisfies all existing constraints is called a *feasible solution*. While the set of all feasible solutions is called a feasible region or also called a Pareto set. Pareto optimal front solution or Pareto front is a set of optimal objective function values. One method used to solve multi-objective problems is the weighted sum method [7].

**Multi-Objective Portfolio Optimization Mean Variance Model**

In portfolio optimization with a multi-objective approach, there are two functions to be optimized, namely minimizing risk and maximizing portfolio returns. The multi-objective problem in the mean variance model is as follows:

Minimize $V = y^T Q y$ (5)
Maximize $R = r^T y$ (6)
Constraint $e^T y = 1$ (7)
$z_i y_{min} \leq y_i \leq z_i y_{max}, i = 1,2, \dots, n$ (8)
$z_i \in \{0,1\},, i = 1,2, \dots, n$ (9)
$\sum_{i=1}^{n} z_i = K$ (10)

where: $V$ = Risk.
$y$ = The proportion of investment in stocks.

$$Q = \begin{pmatrix} \sigma_{11} & \sigma_{12} & \cdots & \sigma_{1n} \\ \sigma_{21} & \sigma_{22} & \cdots & \sigma_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{n1} & \sigma_{n2} & \cdots & \sigma_{nn} \end{pmatrix}$$

$R$= Expected Return
$K$ = The number of stocks purchased.

**Multi-Objective Portfolio Optimization Semi-variance Model**

Portfolio optimization aims to select different proportions of stocks (or other market assets) to combine in a portfolio with the aim of maximizing the overall *expected return* and minimizing the overall risk. In financial markets, there is a trade-off between *return* and risk. In general, the better the return we get (higher return), the worse the risk we get (higher risk) [8-10]. Different combinations of asset weights (the proportion of the amount allocated to each asset with respect to the amount of money available) give different outcomes regarding expected return and risk. In general, portfolio optimization problems use variance (or standard deviation) as a measure of risk. Although generally accepted, this measure is not the most appropriate for assessing risk, because it considers both bad (below average) and good (above average) deviations. However, as Markowitz recognized, investors only care about adverse variances. In this context, Markowitz proposed an alternative risk measure, semi-variance [11], which only takes into account adverse deviations. Chang defines:

Minimize $\sum_{t=1, r_t<\bar{r}}^{T} (r_t - \bar{r})^2 / T$ (11)
Maximize $\bar{\tau}$ (12)
Constraint $e^T y = 1$ (13)

$$z_i y_{min} \leq y_i \leq z_i y_{max}, i = 1,2, \dots, n \quad (14)$$

$$z_i \in \{0,1\},, i = 1,2, \dots, n \quad (15)$$

$$r_t = \log_e \left[ \left( \sum_{i=1}^{n} y_i \, V_{it} / V_{iT} \right) \left( \sum_{i=1}^{n} y_i V_{it-1} / V_{iT} \right) \right], t = 1, \dots, T \quad (16)$$

$$z_i = K \quad (17)$$

$$\sum_{t=1}^{T} r_t / T \quad (18)$$

Where: $r_t$ = return to t

$\bar{r}$ = mean return

$V_{iT}$= stock price , $i = 1,2, \dots, n$ at time, $t = 1, \dots, T$.

### 3.2 Spiral Optimization Method

Tamura and Yasuda introduced the spiral optimization method [12], a metaheuristic method based on the analogy of natural phenomena, such as whirlpools and snail shells.

**2-Dimensional Spiral Model**

A point $[x_1(k), x_2(k)]$ in two-dimensional coordinates is rotated about the origin by θ counterclockwise to obtain $[x_1(k+1), x_2(k+1)]$ where k is a positive integer with the equation:

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = R_{1,2}^{(2)} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} \quad (19)$$

To produce a spiral model that produces a point that converges to the centre point (0,0), the rotation matrix $R_{1,2}^{(2)}$ is multiplied by $r$, which is the conversion rate of the distance between the point and the centre point, with $0 < r < 1$ .

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = rR_{1,2}^{(2)} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} \quad (20)$$

Equation (20) applies to the spiral model with the centre point (0,0); the following is a spiral model with the centre point $(x^*)$ arbitrary:

$$x(k+1) - x^* = S_2(r, \theta)(\mathrm{x}(k) - x^*)$$

$$x(k+1) = S_2(r, \theta)(x(k) - x^*) + x^*$$

$$x(k+1) = S_2(r, \theta)x(k) - (S_2(r, \theta) - I_2)x^* \quad (21)$$

**3-Dimensional Spiral Model**

The 3-dimensional spiral model is the same as equation (21) but the rotation matrix is different. To rotate a point in the 3-dimensional plane, the following rotation matrix is required:

$$R_{1,2}^{(3)}(\theta) \begin{bmatrix} cos\,\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

$$R_{1,3}^{(3)} \begin{bmatrix} cos\,\theta & 0 & -\sin\theta \\ \sin\theta & 1 & 0 \\ 0 & 0 & \cos\theta \end{bmatrix},$$

$$R_{2,3}^{(3)} \begin{bmatrix} 1 & 0 & 0 \\ 0 & cos\,\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$$

The rotation matrix of the 3-dimensional spiral model that rotates the point x in the plane $(x_1, x_2, x_3)$ is the multiplication of the 3 matrices above, namely:

$$R^{(3)}\,(\theta) = R^{(3)}_{2,3}(\theta) \times R^{(3)}_{1,3}(\theta) \times R^{(3)}_{1,2}(\theta)$$

***n*-Dimensional Spiral Model**

Based on the 2-dimensional and 3-dimensional spiral optimization construction, the rotation matrix for the n-dimensional spiral model is:

$$R^{(n)}\,(\theta) = R^{(n)}_{n-1,n}(\theta) \times R^{(n)}_{n-2,n}(\theta) \times \dots \times R^{(n)}_{2,3}(\theta) \times R^{(n)}_{1,n}(\theta) \times \dots \times R^{(n)}_{1,3}(\theta) \times R^{(n)}_{1,2}(\theta)$$

$$R^{(n)}\,(\theta) = \prod_{i=1}^{n-1}\left(\prod_{j=1}^{i} R^{(n)}_{n-1,n+1-j}\,(\theta)\right) \tag{22}$$

where

$$R^{n}_{i,j}(\theta_{i,j}) = \begin{array}{c} \\ \\ \\ i \\ \\ \\ \\ j \\ \\ \\ \\ \end{array} \begin{bmatrix} 1 & & & & & & & & & & \\ & \ddots & & & & & & & & & \\ & & 1 & & & & & & & & \\ & & & \cos\theta_{i,i} & & & & -\sin\theta_{i,j} & & & \\ & & & & 1 & & & & & & \\ & & & & & \ddots & & & & & \\ & & & & & & 1 & & & & \\ & & & \sin\theta_{j,i} & & & & \cos\theta_{j,j} & & & \\ & & & & & & & & 1 & & \\ & & & & & & & & & \ddots & \\ & & & & & & & & & & 1 \end{bmatrix}$$

with the empty matrix elements having zero values. The n-dimensional spiral model formula with the centre point ($x^*$):

$$x(k+1) = S_n(r,\theta)x(k) - (S_n(r,\theta) - I_n)x^* \quad (23)$$

where $S_n(r,\theta) = rR^{(n)}(\,\theta)$ and $I_n$ identity matrix $n \times n$.

### 3.3 Spiral Optimization Algorithm

Spiral optimization algorithm:

1) Specify the number of search points
$$m \geq 2, 0 \leq \theta < 2\pi, 0 < r < 1$$
from $S_n(r,\theta)$ and maximum iteration $k_{max.}$ choose $k = 0$.
2) Generate search points $x_i(0) \in \mathbb{R}^n, i = 1,2,\dots,m$ in the feasible area and determine the centre $x^*$ with $x_i(0) = x_{ig}(0)$, $i_g = arg\ min_i f\ (x_i(0)), i = 1,2,\dots,m$.
3) update $x_i$.

$$x(k+1) = S_n(r,\theta)x(k) - (Sn(r,\theta) - I_n)x^*, i = 1,2,\dots,m$$

4) Update $x^*$

$$x^* = x_{ig}(k+1), i_g = arg\ min_i f(x_i(k+1)), i = 1,2,\dots,m.$$

5) $k = k + 1$. Jika $k = kmax$ then stop. If not choose $k = k + 1$ back to process 3.

## 4 Results and Discussion

The data used is Hangseng stock data. The Hangseng stock data was taken monthly between 5 May 2012 and 5 May 2017, totalling 49 stocks with the following mean returns.

**Table 1.** Mean return data from 49 Hang Seng stocks

| Saham | Mean Return | Saham | Mean Return |
|---|---|---|---|
| 1 | 0.00288 | 16 | 0.01101 |
| 2 | 0.00950 | 17 | 0.03156 |
| 3 | 0.00893 | 18 | 0.00064 |
| 4 | 0.01300 | 19 | 0.01388 |
| 5 | 0.00357 | 20 | -0.00252 |
| 16 | -0.00252 | 41 | -0.00464 |
| 17 | 0.00615 | 42 | -0.00326 |
| 18 | 0.00415 | 43 | -0.01309 |
| 19 | 0.00338 | 44 | -0.0070 |
| 20 | 0.00758 | 45 | -0.00139 |
| 21 | 0.00872 | 46 | -0.00189 |
| 22 | 0.00412 | 47 | -0.01005 |
| 23 | -0.00518 | 48 | 0.00579 |
| 24 | 0.0019 | 49 | 0.02262 |
| 25 | 0.00213 | | |

### 4.1 Results of Spiral Implementation for Multi-Objective Portfolio

**Multi-Objective for Mean-Variance Risk Measure**

The lower limit for each proportion $y_{min} = 0.05$ and the upper limit for the proportion $y_{max} = 1$ are set. The weights used are 21 weights, namely $0 \leq w_1 \leq 1$ with an interval for each weight $w_1 = 0.05$ and $w_2 = 1 - w_1$. One set of weights produces one pair of V and R values . Where V is a function that minimizes risk and R is a function that maximizes return based on equations (5) and (6), so that for the 21 pairs of weights that have been set, 21 pairs of optimal V and R values are obtained which can form the Pareto Optimal Front.

1) Hang Seng Data

Portfolio optimization simulation for a multi-objective mean-variance case using 30 Hang Seng stocks from 49 Hang Seng stocks in Table 1. Six abnormal stocks and 24 other stocks were selected randomly.

- K=5

The parameters used are m = 3000; k = 3000; θ = π / 4; r = 0:997; ρ = 10 ^ 5. The Pareto front formed from 21 pairs of weights produces a combination of risk (σ) and return (R) pairs. Pareto front data Hangseng can be seen in Figure 1. The total time required for one run is 3306.56 seconds. The results of the optimization of data Hangseng K = 5 can be seen in Table 2.

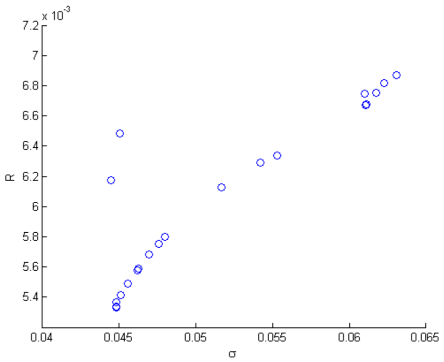


**Fig 1**. Pareto front K=5 mean variance Hangseng

**Table 2**. Results of Mean Variance Hangseng for K=5

| $w_1$ | 0,1 | | 0,5 | | 0,9 | |
|---|---|---|---|---|---|---|
| $w_2$ | 0,9 | | 0,5 | | 0,1 | |
| Saham | $y_i$ | $z_i$ | $y_i$ | $z_i$ | $y_i$ | $z_i$ |
| 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | 0 | 0 | 0 | 0 | 0 | 0 |
| 3 | 0 | 0 | 0 | 0 | 0 | 0 |
| 4 | 0,2153 | 1 | 0,0827 | 1 | 0,0571 | 1 |
| 5 | 0 | 0 | 0 | 0 | 0 | 0 |
| 6 | 0 | 0 | 0 | 0 | 0 | 0 |
| 7 | 0 | 0 | 0 | 0 | 0 | 0 |
| 8 | 0 | 0 | 0 | 0 | 0 | 0 |
| 9 | 0,0500 | 1 | 0,0500 | 1 | 0,0500 | 1 |
| 10 | 0 | 0 | 0 | 0 | 0 | 0 |
| 11 | 0,0500 | 1 | 0,6222 | 1 | 0,7747 | 1 |
| 12 | 0 | 0 | 0 | 0 | 0 | 0 |
| 13 | 0 | 0 | 0 | 0 | 0 | 0 |
| 14 | 0,0501 | 1 | 0,0500 | 1 | 0,0682 | 1 |
| 15 | 0 | 0 | 0 | 0 | 0 | 0 |
| 16 | 0,6347 | 1 | 0,1951 | 1 | 0,0500 | 1 |
| 17 | 0 | 0 | 0 | 0 | 0 | 0 |
| 18 | 0 | 0 | 0 | 0 | 0 | 0 |
| 19 | 0 | 0 | 0 | 0 | 0 | 0 |
| 20 | 0 | 0 | 0 | 0 | 0 | 0 |
| 21 | 0 | 0 | 0 | 0 | 0 | 0 |
| 22 | 0 | 0 | 0 | 0 | 0 | 0 |
| 23 | 0 | 0 | 0 | 0 | 0 | 0 |
| 24 | 0 | 0 | 0 | 0 | 0 | 0 |
| 25 | 0 | 0 | 0 | 0 | 0 | 0 |
| 26 | 0 | 0 | 0 | 0 | 0 | 0 |
| 27 | 0 | 0 | 0 | 0 | 0 | 0 |
| 28 | 0 | 0 | 0 | 0 | 0 | 0 |
| 29 | 0 | 0 | 0 | 0 | 0 | 0 |
| 30 | 0 | 0 | 0 | 0 | 0 | 0 |
| V | 0.0038 | | 0.0023 | | 0.0020 | |
| $\sigma$ | 0.0617 | | 0.0480 | | 0.0445 | |
| R | 0.0068 | | 0.0058 | | 0.0062 | |

- K=10

The parameters used are m = 3000; k = 3000; θ = π / 4; r = 0:997; ρ = 10 ^ 5. The Pareto front formed from 21 pairs of weights produces a combination of risk (σ) and return (R) pairs. The Pareto front of the Hangseng data can be seen in Figure 1. The total time required for one run is 2959.64 seconds. The results of optimising the Hangseng data K = 10 can be seen in Table 3.

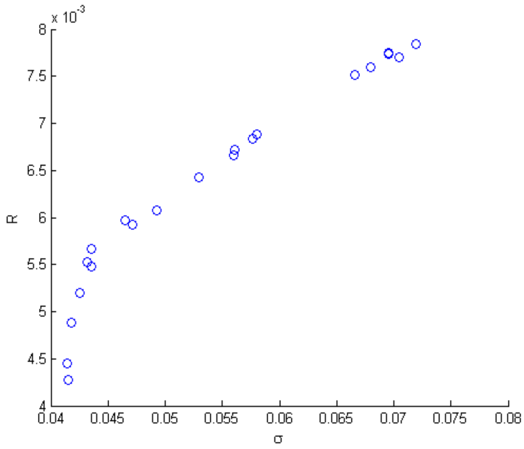


**Fig 2**. Pareto front K=10 mean variance Hangseng

**Table 3.** Results of Mean Variance Hangseng for *K*=10

| $w_1$ | 0,1 | | 0,5 | | 0,9 | |
|---|---|---|---|---|---|---|
| $w_2$ | 0,9 | | 0,5 | | 0,1 | |
| Saham | $y_i$ | $z_i$ | $y_i$ | $z_i$ | $y_i$ | $z_i$ |
| 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | 0 | 0 | 0 | 0 | 0 | 0 |
| 3 | 0 | 0 | 0 | 0 | 0 | 0 |
| 4 | 0 | 0 | 0 | 0 | 0 | 0 |
| 5 | 0,0500 | 1 | 0,0500 | 1 | 0,0828 | 1 |
| 6 | 0 | 0 | 0 | 0 | 0 | 0 |
| 7 | 0 | 0 | 0 | 0 | 0 | 0 |
| 8 | 0,1737 | 1 | 0,0984 | 1 | 0,0502 | 1 |
| 9 | 0,0500 | 1 | 0,0501 | 1 | 0,0500 | 1 |
| 10 | 0 | 0 | 0 | 0 | 0 | 0 |
| 11 | 0 | 0 | 0 | 0 | 0 | 0 |
| 12 | 0 | 0 | 0 | 0 | 0 | 0 |
| 13 | 0,0979 | 1 | 0,2148 | 1 | 0,0501 | 1 |
| 14 | 0 | 0 | 0 | 0 | 0 | 0 |
| 15 | 0 | 0 | 0 | 0 | 0 | 0 |
| 16 | 0,0784 | 1 | 0,2148 | 1 | 0,0501 | 1 |
| 17 | 0 | 0 | 0 | 0 | 0 | 0 |
| 18 | 0 | 0 | 0 | 0 | 0 | 0 |
| 19 | 0 | 0 | 0 | 0 | 0 | 0 |
| 20 | 0 | 0 | 0 | 0 | 0 | 0 |
| 21 | 0 | 0 | 0 | 0 | 0 | 0 |
| 22 | 0,0500 | 1 | 0,0500 | 1 | 0,0500 | 1 |
| 23 | 0,0500 | 1 | 0,0500 | 1 | 0,0500 | 1 |
| 24 | 0,3500 | 1 | 0,1022 | 1 | 0,0500 | 1 |
| 25 | 0 | 0 | 0 | 0 | 0 | 0 |
| 26 | 0,0501 | 1 | 0,0500 | 1 | 0,0502 | 1 |
| 27 | 0 | 0 | 0 | 0 | 0 | 0 |
| 28 | 0,0500 | 1 | 0,0500 | 1 | 0,1322 | 1 |
| 29 | 0 | 0 | 0 | 0 | 0 | 0 |
| 30 | 0 | 0 | 0 | 0 | 0 | 0 |
| V | 0.0048 | | 0.0028 | | 0.0017 | |
| $\sigma$ | 0.0695 | | 0.0529 | | 0.0418 | |
| R | 0.0077 | | 0.0064 | | 0.0049 | |

Figures 1 and 2 show that the greater the value of K desired by an investor, the shorter the resulting Pareto front.

### 4.2 Multi-Objective for Semi-Variance Risk Measure

Portfolio optimization simulation for multi-objective semi-variance case using 30 Hang Seng stocks from 49 Hang Seng stocks in Table 1.

- K=5

The parameters used are m = 3000; k = 3000; θ = π / 4; r = 0:997; ρ = 10 ^ 5. The Pareto front formed from 21 pairs of weights produces a combination of risk (σ) and return (R) pairs. Pareto front data Hangseng can be seen in Figure 3. The total time required for one run is 9334.30 seconds. The results of the optimization of data Hangseng K = 5 can be seen in Table 4.

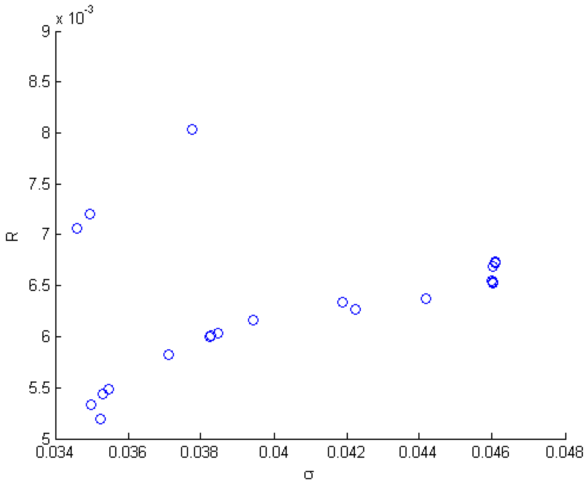


**Fig 3.** Pareto front K=5 semi variance Hangseng

**Table 4**. Semi Variance Hangseng Results for *K*=5

| $w_1$ | 0,1 | | 0,5 | | 0,9 | |
|---|---|---|---|---|---|---|
| $w_2$ | 0,9 | | 0,5 | | 0,1 | |
| Saham | $y_i$ | $z_i$ | $y_i$ | $z_i$ | $y_i$ | $z_i$ |
| 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | 0 | 0 | 0 | 0 | 0 | 0 |
| 3 | 0 | 0 | 0 | 0 | 0 | 0 |
| 4 | 0,0500 | 1 | 0,0501 | 1 | 0,1517 | 1 |
| 5 | 0 | 0 | 0 | 0 | 0 | 0 |
| 6 | 0 | 0 | 0 | 0 | 0 | 0 |
| 7 | 0 | 0 | 0 | 0 | 0 | 0 |
| 8 | 0 | 0 | 0 | 0 | 0 | 0 |
| 9 | 0,0500 | 1 | 0,0500 | 1 | 0,0500 | 1 |
| 10 | 0 | 0 | 0 | 0 | 0 | 0 |
| 11 | 0,7998 | 1 | 0,4555 | 1 | 0,0500 | 1 |
| 12 | 0 | 0 | 0 | 0 | 0 | 0 |
| 13 | 0 | 0 | 0 | 0 | 0 | 0 |
| 14 | 0,0502 | 1 | 0,0500 | 1 | 0,0500 | 1 |
| 15 | 0 | 0 | 0 | 0 | 0 | 0 |
| 16 | 0,0500 | 1 | 0,3945 | 1 | 0,6893 | 1 |
| 17 | 0 | 0 | 0 | 0 | 0 | 0 |
| 18 | 0 | 0 | 0 | 0 | 0 | 0 |
| 19 | 0 | 0 | 0 | 0 | 0 | 0 |

| | | | | | | |
|---|---|---|---|---|---|---|
| 20 | 0 | 0 | 0 | 0 | 0 | 0 |
| 21 | 0 | 0 | 0 | 0 | 0 | 0 |
| 22 | 0 | 0 | 0 | 0 | 0 | 0 |
| 23 | 0 | 0 | 0 | 0 | 0 | 0 |
| 24 | 0 | 0 | 0 | 0 | 0 | 0 |
| 25 | 0 | 0 | 0 | 0 | 0 | 0 |
| 26 | 0 | 0 | 0 | 0 | 0 | 0 |
| 27 | 0 | 0 | 0 | 0 | 0 | 0 |
| 28 | 0 | 0 | 0 | 0 | 0 | 0 |
| 29 | 0 | 0 | 0 | 0 | 0 | 0 |
| 30 | 0 | 0 | 0 | 0 | 0 | 0 |
| V | 0.0021 | | 0.0015 | | 0.0012 | |
| $\sigma$ | 0.046 | | 0.0382 | | 0.0353 | |
| R | 0.0065 | | 0.006 | | 0.0054 | |

- K=10

The parameters used are m = 3000; k = 3000; θ = π / 4; r = 0:997; ρ = 10 ^ 5. The Pareto front formed from 21 pairs of weights produces a combination of risk (σ) and return (R) pairs. The Pareto front of the Hangseng data can be seen in Figure 4. The total time required for one run is 8938.53 seconds. The results of optimising the Hangseng data K = 10 can be seen in Table 5. Figures 3 and 4 show that the greater the value of K desired by an investor, the shorter the Pareto front.

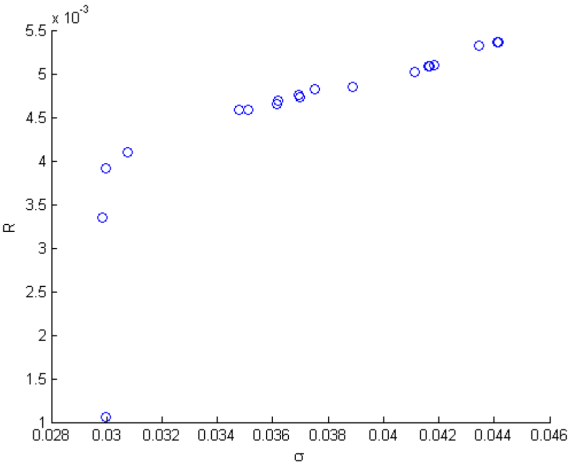


**Fig 4.** Pareto front K=10 semi variance Hangseng

**Table 5.** Semi Variance Hangseng Results for K=10

| $w_1$ | 0,1 | | 0,5 | | 0,9 | |
|---|---|---|---|---|---|---|
| $w_2$ | 0,9 | | 0,5 | | 0,1 | |
| Saham | $y_i$ | $z_i$ | $y_i$ | $z_i$ | $y_i$ | $z_i$ |
| 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | 0 | 0 | 0 | 0 | 0 | 0 |
| 3 | 0 | 0 | 0 | 0 | 0 | 0 |
| 4 | 0 | 0 | 0 | 0 | 0 | 0 |
| 5 | 0,0741 | 1 | 0,0500 | 1 | 0,0500 | 1 |
| 6 | 0 | 0 | 0 | 0 | 0 | 0 |
| 7 | 0 | 0 | 0 | 0 | 0 | 0 |
| 8 | 0,0503 | 1 | 0,1554 | 1 | 0.1389 | 1 |
| 9 | 0,0500 | 1 | 0,0500 | 1 | 0,0500 | 1 |
| 10 | 0 | 0 | 0 | 0 | 0 | 0 |
| 11 | 0 | 0 | 0 | 0 | 0 | 0 |
| 12 | 0 | 0 | 0 | 0 | 0 | 0 |

| | | | | | | |
|---|---|---|---|---|---|---|
| 13 | 0.4196 | 1 | 0.2216 | 1 | 0.1166 | 1 |
| 14 | 0 | 0 | 0 | 0 | 0 | 0 |
| 15 | 0 | 0 | 0 | 0 | 0 | 0 |
| 16 | 0.1067 | 1 | 0.2144 | 1 | 0.1775 | 1 |
| 17 | 0 | 0 | 0 | 0 | 0 | 0 |
| 18 | 0 | 0 | 0 | 0 | 0 | 0 |
| 19 | 0 | 0 | 0 | 0 | 0 | 0 |
| 20 | 0 | 0 | 0 | 0 | 0 | 0 |
| 21 | 0 | 0 | 0 | 0 | 0 | 0 |
| 22 | 0.0500 | 1 | 0.0500 | 1 | 0.0500 | 1 |
| 23 | 0.0500 | 1 | 0.0500 | 1 | 0.0500 | 1 |
| 24 | 0.0500 | 1 | 0.1085 | 1 | 0.2669 | 1 |
| 25 | 0 | 0 | 0 | 0 | 0 | 0 |
| 26 | 0.0501 | 1 | 0.0500 | 1 | 0.0500 | 1 |
| 27 | 0 | 0 | 0 | 0 | 0 | 0 |
| 28 | 0.0991 | 1 | 0.0500 | 1 | 0.0500 | 1 |
| 29 | 0 | 0 | 0 | 0 | 0 | 0 |
| 30 | 0 | 0 | 0 | 0 | 0 | 0 |
| V | 0.0020 | | 0.0013 | | 0.0009 | |
| $\sigma$ | 0.0442 | | 0.0362 | | 0.0300 | |
| R | 0.0054 | | 0.0046 | | 0.0039 | |

As we know, in mean-variance, returns are assumed to be normally distributed, and investors are assumed to be risk-averse. The semi-variance risk measure does not require any distribution, and the risk part of the mean-variance can be replaced with semi-variance. Optimizing the mean-variance and semi-variance risk measures shows that semi-variance provides a more negligible risk than the mean-variance for each K in Hang Seng stocks. This is very important for an investor who prioritizes minimizing risk, so semi-variance is very suitable because the results also provide a more negligible risk than the mean-variance. However, the computation for semi-variance takes longer than mean-variance; this is what makes semi-variance less effective to use.

## 5 Conclusion

Spiral models with simple cases that do not involve high dimensions can be completed quickly. However, the algorithm in this Matlab programming runs slowly for high dimensions with significant maximum spread points and iterations, so in terms of computation, this is not effective. Although the spiral method can solve multi-objective portfolio optimization, it is suspected that some solutions are still trapped in local minima, or the results obtained have not converged, resulting in the resulting Pareto front not being optimal.

Markowitz assumes that portfolio returns follow a multivariate normal distribution in multi-objective mean-variance optimisation. However, almost all portfolio returns do not follow a multivariate normal distribution, as seen in the Hangseng indices. Therefore, in cases where the data does not follow a multivariate normal distribution, it is not recommended to perform analysis using a multivariate normal distribution, as the analysis will be incomplete and inaccurate. Hence, this research addresses multi-objective mean-variance and multi-objective optimization with semivariance, which does not require returns to follow a normal distribution.

In the future, modifications can be made to the spiral optimization method so that it can solve optimization problems with high dimensions, thus eliminating the need to use large initial spread points and maximum iterations, which require much time to execute the program. Also, understanding the relationship between spiral parameters so that in solving high-dimensional problems, they can be easily determined, and the optimization time is shorter.

## Acknowledgement

I want to express my special gratitude to my supervisor, Prof. Dr. Kuntjoro Adji Sidarto, for his guidance, input, and dedication in helping me complete this research. I also thank my family, friends, and all parties who have directly or indirectly provided encouragement and assistance during this research process.

May all the support given be a blessing and goodness for all of us. Hopefully, this research can contribute, both directly and indirectly, to the business world, especially in the investment sector, by providing useful insights and becoming a basis for more effective decision-making. It is also expected to encourage the development of innovative and sustainable investment strategies and provide practical benefits for business actors and related stakeholders.

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