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Optimal Portfolio Risk Estimation Using Expected Shortfall of Jakarta Islamic Index (JII) Shares

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Abstract. Forming an optimal portfolio using the Mean-Variance method with Downside Deviation as a measure of risk produces a good combination of assets. Before investing, estimating risk as a worst-case scenario is very important. Expected shortfall (ES) serves as a risk measure that takes into account the possibility of losses that exceed Value at Risk (VaR). This study aims to determine the optimal portfolio and compare ES and VaR at the 90%, 95%, and 99% confidence levels. This research data involves 3 stocks namely ACES, WIFI, and TLKM. Based on the results of the analysis conducted, the optimal combination of weights is ACES (19%), WIFI (10%), and TLKM (71%). Comparison of ES and VaR shows that the higher the level of confidence, the higher the VaR and ES values generated, so the greater the risk that will be borne by investors and the capital allocation used to cover these losses.

Keywords: Optimal Portfolio, Downside Deviation, VaR, Expected Shortfall

1 Introduction

Investment is essentially the placement of several funds at this time with the hope of obtaining future profits [1]. Some types of instruments used in investing include stocks, bonds, gold, and property. The instrument that is most in demand by the Indonesian people is investment in stocks [2]. The Indonesia Stock Exchange launched the Jakarta Islamic Index (JII) which is an index of the most liquid stocks and meets sharia criteria following the provisions of the DSN (National Sharia Council). In investing in the capital market, investors usually form an optimal portfolio to help investors make good investment decisions. So in the formation of a stock portfolio, it is necessary to form diversification in investing, which means that investors must form a portfolio by selecting several assets to reduce risk but still generate expected profits[3].

The method often used in portfolio optimization efforts is mean-variance (MV).[4] There are several weaknesses of MV, including the use of variance as risk. The shortcomings of MV gave rise to the Downside Deviation (DD) model which assumes the standard deviation of returns located below the target (benchmark) as a measure of risk[5]. The optimal portfolio has risks because stocks can have high-risk and high-return characteristics. This means that stocks are securities that not only offer high profit opportunities but also have high risks [6]. Therefore, before deciding to invest in the optimal portfolio, measuring or estimating the risk in the portfolio is very important in financial analysis, because every business has an equal challenge between revenue growth and risk management. So a measurement tool is needed to estimate the risk so that it can be known to what extent investors can safely invest. The most commonly applied risk measurement is Value at Risk (VaR) [7]. The VaR method is an international standard for measuring financial risk, but VaR also has the disadvantage that VaR does not pay attention to any losses that exceed the VaR level and VaR does not fulfill the nature of subadditivity. So VaR cannot reflect diversification by forming a portfolio that aims to minimize risk [8]. To overcome this problem, Expected Shortfall (ES) can be used.

In general, ES defines an expected risk measure whose value is above VaR. The ES method is an estimation or estimation of risk that can work on data that is normally or abnormally distributed[9]. The ES method has sub-additive and convex properties. The sub-additive property indicates that the ES of a portfolio consisting of two assets is smaller or equal to the sum of the ES of each asset, which allows ES to accurately reflect the diversification effect because the purpose of diversification is to reduce risk. The convex nature of ES means

that it can be used in optimization techniques. These properties are the advantages of ES compared to VaR. In addition, the advantage of ES over VaR is that ES is a risk measure that takes into account the possibility of losses that exceed the VaR value [10]. Based on this description, this research is titled "Estimation of Optimal Portfolio Risk Using Expected Shortfall of Jakarta Islamic Index (JII) Shares".

2 Theoretical Basic

2.1 Stock Return, Expected Return, and Stock Return Covariance

Return is the result (rate of return) obtained as a result of the investment made, mathematically a geometric approach can be used. This approach uses the logarithm of the price ratio, mathematically expressed as follows [10]:

$$R_{it} = In\frac{p_{it}}{p_{i(t-1)}}\ , \quad i = 1,2,\dots,k \tag{1}$$

Systematically, the expected return formulation can be written using the equation [11]

$$E(R_i) = \frac{\sum_{t=1}^{T} R_{it}}{T}\ , \qquad i = 1,2,\dots,k \tag{2}$$

The DD model carries the idea that the value of stock returns that are below a predetermined value (benchmark) will be considered a risk. [4] termed the concept of downside risk as DD, in the following equation:

$$DD_i = \sqrt{\frac{\sum_{t=1}^{T}\left(min(R_{it}-b,0)\right)^2}{T-1}}\ , i = 1,2,\dots,k \tag{3}$$

The above equation shows that the benchmark value (b) uses the daily average Bank Indonesia interest rate where the rate of return above the value of (b) is not taken into account as a risk or equal to 0, while the rate of return below the value of (b) will increase the covariance value which means increasing the risk factor. The formation of the DD model efficient portfolio is almost the same as the MV method, the difference lies in the calculation of the variance-covariance matrix. The calculation of the covariance of the DD model is expressed by equation [4]:

$$\sigma_{ij} = \sum_{t=1}^{T} \frac{min[R_{it}-b,0]\ min\left[R_{jt}-b,0\right]}{T-1} \tag{4}$$

2.2 Optimal Portfolio Using Mean-Variance

A portfolio is a combination of several assets or instruments formed by investors to obtain future returns[12].The essence of portfolio formation is to minimize or reduce risk as little as possible by allocating funds to various investment alternatives, known as diversification [13]. The expected return portfolio is a weighted average of each expected return of each stock in the portfolio. This can be calculated using the equation:

$$E\left(R_p\right) = \sum_{i=1}^{n} W_i\ E(R_i) \tag{5}$$

One way to measure risk is by standard deviation, which is the square root of the variance of the security's return value. Thus, the standard deviation of portfolio returns, which is the portfolio risk, can be written as follows:

$$\sigma_p^2 = \sum_{i=1}^{n} w_i^2\sigma_i^2 + \sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j \sigma_{ij}\ , \quad i \neq j \tag{6}$$

$$\sigma_P = \sqrt{\sum_{i=1}^{n} w_i^2 \sigma_i^2 + \sum_{i=1}^{n} \sum_{j=1}^{n} w_i w_j \sigma_{ij}} \quad , i \neq j \tag{7}$$

The variance-covariance matrix is formed based on equations (3) and (4) and then expressed by equation (8).

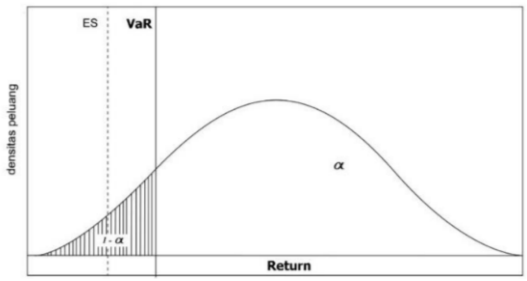
$$\boldsymbol{\Sigma} = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} & \dots & \sigma_{1n} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} & \dots & \sigma_{25} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} & \dots & \sigma_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \sigma_{n1} & \sigma_{n2} & \sigma_{n3} & \dots & \sigma_{nn} \end{bmatrix} \tag{8}$$

The weight of each stock in the optimal portfolio with minimum risk value can be calculated using the equation

$$\boldsymbol{w} = \frac{\boldsymbol{\Sigma^{-1}} \mathbf{1}_N}{\mathbf{1}_N^T \boldsymbol{\Sigma^{-1}} \mathbf{1}_N} \tag{9}$$

2.3 Value at Risk (VaR) and Expected Return (ES)

VaR can be defined as an estimate of the maximum loss achieved during a certain period under normal market conditions at a certain confidence level [7]: Three main methods are often used to estimate VaR, namely historical simulation, variance-covariance, and Monte Carlo methods [14]. VaR calculation with variance-covariance method is a VaR calculation with a parametric approach:



Source: Kusumawardani at al., 2019

Figure 1. VaR and ES for confidence levels $(1-\alpha)$

VaR at confidence level is the lower bound value of the return where the return will not be smaller than this lower bound with a confidence level of. VaR serves to determine "How much is the investor's expected loss (in percentage or amount) during the investment period. with a confidence level of "[15]. By using the period conversion rule, the calculation of VaR with the confidence level of in period can be expressed as follows:

$$VaR_{(1-\alpha)} = \mu + \Phi^{-1}(\alpha)\, \sigma\, \sqrt{t} \tag{10}$$

The Expected Shortfall (ES) method means the amount of loss that will be borne if there is a loss that exceeds VaR. In Figure 1, ES is marked with a shaded line that is below the VaR value. ES can overcome the weakness of VaR because VaR does not pay attention to any losses that exceed the VaR level. Meanwhile, ES can take into account losses above the VaR value that may occur. Therefore, ES is used as an alternative in risk measurement that serves to overcome the problems that occur in VaR. ES At the confidence level in a period period as follows:

$$ES_{1-\alpha} = \mu_p - \sigma_p \frac{\Phi(VaR_{(1-\alpha)})}{\alpha} \sqrt{t} \tag{11}$$

3. METHODS

This type of research is applied research and the data used is secondary data. The secondary data source used is a list of daily closing prices of shares and the volume of sales of shares listed on the Jakarta Islamic Index (JII) stock index in the period January 1, 2023 - December 31, 2023 from the Indonesia Stock Exchange Investment Gallery, Faculty of Economics UNP and the official website of the Indonesia Stock Exchange (IDX), namely https://finance.yahoo.com/ , and for the risk-free rate taken from www.bi.co.id.

The steps taken in analyzing the data in this study are:

1. The procedure for finding the optimal portfolio uses MV with DD as a measure of risk:
   a. Collecting stocks that are included in JII stocks for 1 year, namely the period January 1 - December 31, 2023.
   b. Select 10 stocks that have the highest average sales volume of JII stocks for the period January 1 - December 31, 2023.
   c. Calculating the return value of historical data of the daily closing price of 10 stocks using equation (1)
   d. Determining the expected value of stock returns using equation (2)
   e. Calculating the Downside Deviation value using equation (3)
   f. Calculate the covariance value based on the downside deviation method of the selected stocks using equation (4).
   g. Form a variance-covariance matrix based on the downside deviation method using equation (8).
   h. Calculate the weight value of each stock in the optimal portfolio with the minimum risk value based on equation (9).
   i. Determining the expected return of the portfolio using equation (5)
   j. Determine portfolio risk using equation (6)
2. The procedure for calculating the optimal portfolio risk estimate using the Expected Shortfall
   a. Calculating the VaR value at the confidence interval of the equation $(1-\alpha)$ using the variance-covariance method in equation (10)
   b. Calculating the Expected Shortfall value using equation (11)
   c. Comparing VaR and ES at confidence intervals $(1-\alpha)$

4. RESULT AND DISCUSSION

The data used in this study are daily closing stock price data on companies listed in the JII index, and the BI-7 days repo rate in the period January 1 - December 31, 2023. Sample selection on the JII index is by collecting 30 stocks. Furthermore, selecting 10 stocks from 30 stocks incorporated in the JII. The average value of the 10 stocks that have the highest sales volume can be seen in Table 1.

Table 1. JII Stocks with the Highest Average Sales Volume

NO	STOCK	VOL (Lot)
1	GOTO	4442750407.98319
2	BRMS	314971279.411765
3	WIFI	167910561.764706
4	ACES	92848340.7563025
5	TLKM	90569952.5210084
6	ADRO	60154467.2268908
7	ANTM	51342691.5966387
8	MDKA	50125844.9579832
9	PGAS	48714860.5042017
10	ADMR	48064136.9747899

First, calculate the return value of each stock forming the portfolio based on daily closing price data using equation (1). The expected return value can be seen in Table 2.

Table 2. Expected Return Value of Each Stock

NO	STOCK	EXPECTED RETURN
1	GOTO	−0.000328791
2	BRMS	0.000254725
3	WIFI	0.000744287
4	ACES	0.00144891
5	TLKM	0.000162666
6	ADRO	−0.001727108
7	ANTM	−0.000628285
8	MDKA	−0.001785822
9	PGAS	−0.001813729
10	ADMR	−0.000887853

Based on Table 2, it can be seen that the expected return value of the ten stocks forming the portfolio with BRMS, WIFI, ACES, and TLKM shares shows a positive value, meaning that the share price has increased in return value while, GOTO, ADRO, ANTM, MDKA, PGAS, ADMR show a negative value, meaning that the stock is experiencing a downward trend. Next, the benchmark value will be calculated using the daily average Bank Indonesia interest rate in 2023, the benchmark value is obtained. Next, the Downside Deviation (DD) value will be calculated using equation (3), The Downside Deviation value of each stock can be seen in Table 3.

Table 3. Downside Deviation (DD) Value

NO	STOCK	DOWNSIDE DEVIATION
1	GOTO	0.072462749
2	BRMS	0.066448009
3	WIFI	0.064573257
4	ACES	0.063454795
5	TLKM	0.059453435
6	ADRO	0.064653238
7	ANTM	0.061186542
8	MDKA	0.066305682
9	PGAS	0.062546225
10	ADMR	0.068124273

Of the ten stocks with the highest average sales volume above, 6 stocks have negative expected returns, and 4 stocks have positive expected returns. Stocks that have negative expected return values are not included in the calculation, because the possibility of these stocks is not profitable and experiencing losses. Stocks with positive expected return values are worth considering in investing. Stocks that have positive expected return values can be seen in Table 4.

Table 4. Stocks with Positive Expected Return Value

NO	STOCK	EXPECTED RETURN	DOWNSIDEDEVIATION	σ^2
1	ACES	0.001449	0.063455	0.004027
2	WIFI	0.000744	0.064573	0.004170
3	BRMS	0.000255	0.066448	0.004415
4	TLKM	0.000163	0.059453	0.003535

To make it easier to determine efficient stocks, can be seen based on Figure 2.

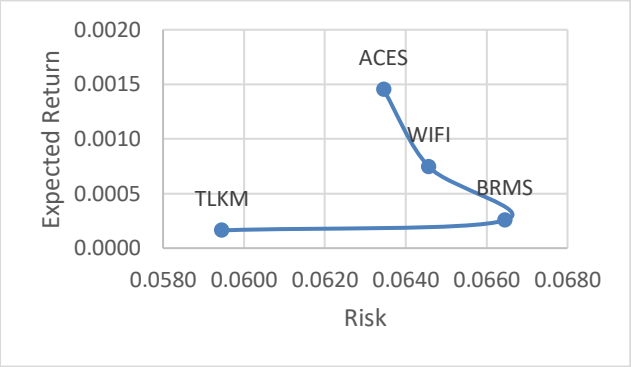


Figure 1. Efficient Stock

The efficient stocks are ACES, WIFI, and TLKM. After knowing the DD value, the covariance value between stocks will be calculated using equation (5). The results are presented in the variance-covariance matrix using equation (8).

$$\Sigma = \begin{bmatrix} 0.00402651 & 0.00339833 & 0.00335647 \\ 0.00339833 & 0.00416971 & 0.00000754 \\ 0.00335647 & 0.00342144 & 0.00353471 \end{bmatrix}$$

Next, the weight of each stock in the portfolio will be calculated using equation (9), The value of each stock can be seen in Table 5.

Table 5. Weight Value of Each Stock

NO	STOCK	W/Weight	W/Weight (%)
1	ACES	0,192414702	19%
2	WIFI	0,096826752	10%
3	TLKM	0,710758545	71%
TOTAL		1	100%

From the weight value above, the expected return value of the portfolio will be obtained using equation (5), the result is obtained0.000466475 and the risk value of the portfolio using the formula in equation (6) obtained a result of $\sigma^2 = 0.00348945$, and the standard deviation value using equation (7) is $\sigma_p = 0.059071536$. As for calculating the value of VaR and ES in estimating the worst possibility of portfolio risk formed is

VaR method used to calculate the risk at the confidence level is with variance-covariance at various significance levels, namely $\alpha = 0.1$, $\alpha = 0.05$, and $\alpha = 0.01$. If $t = 1$ then the VaR value using equation (22). VaR values with various confidence levels can be seen in Table 6.

Table 6. VaR with Various Confidence Levels

TRUST LEVEL $(1-\alpha)$	VaR
99%	−0,136954
95%	−0,096698
90%	−0,075237

The estimated VaR with a minus value indicates a loss of return. From the results of the VaR calculation in Table 6, it can be seen that VaR at the confidence level of 99% produces the highest VaR value among VaR at other confidence levels, namely -0.136954. The interpretation that can be obtained is that at a holding period of 1 day with a confidence level of 1 day 99%, it is obtained that the loss that will occur will not exceed the VaR value, meaning that when investors invest in ACES, WIFI, and TLKM shares there is the confidence that the maximum loss will not exceed the VaR value 99% that the maximum loss will not exceed 14% of the invested funds for a period of one trading day ahead after December 31, 2023. At the confidence level 95% by -0.096698 it means that there is a maximum loss of 10% of the funds invested for a period of 1 day ahead after December 31, 2023. At the confidence level, 90% by -0.075237 means that there is a maximum loss of 8% of funds invested for the period 1 day ahead of December 31, 2023. It can be seen that the greater the level of confidence taken, the greater the risk that must be borne and the capital allocation used to cover these losses.

Expected Shortfall (ES) at various significance levels, namely $\alpha = 0.1$, $\alpha = 0.05$ and $\alpha = 0.01$. If $t = 1$ then by using the formula in equation (11). ES values with various confidence levels can be seen in Table 7.

Table 7. ES with Various Confidence Levels

TRUST LEVEL $(1-\alpha)$	VaR
99%	−0,156972
95%	−0,1211381
90%	−0,103203

Based on Table 7 confidence level 99% obtained an ES value of 16% while the VaR value is 14% The ES value is greater than the VaR value. So it can be interpreted that the biggest loss that an investor will receive for the next 1 day period after December 31, 2023 is the amount of 14% of investment one day after December 31, 2023 and it is still possible if there are worse events such as inflation, economic crises and others experienced by an investor for the next 1 day period from December 31, 2023, then the risk that may occur is with a maximum loss of ES value with a confidence level of 99% i.e.16% At the confidence level 95% obtained ES value of 12% greater than VaR (10%), so that it can be interpreted that the biggest loss that will be received by investors for the next 1 day period is 10% and it is still possible if worse conditions occur, then the risk that may occur is with a maximum loss of ES value with a confidence level of 95% i.e. 12%And at the confidence level 90% obtained ES value of 10% greater than VaR (8%), so it can be interpreted that the biggest loss that investors will receive for the next 1 day period is 8% and it is still possible if worse conditions occur, then the risk that may occur is with a maximum loss of ES value with a confidence level of 90% i.e.10%.

5 CONCLUSION

Based on the objectives of the research and discussion that has been carried out above, it can be concluded that. The optimal portfolio using Mean-Variance with Downside Deviation as a risk measure produces the following optimal weight combination: ACES by 19%, WIFI by 10%, and TLKM by 71% with an expected return of portfolio value of 0.0466%. Comparison between Value at Risk and Expected Shortfall estimated

at three confidence intervals, namely $90\%, 95\%$ and 99% It is found that the higher the level of confidence, the higher the VaR and ES values generated, the greater the risk that will be borne by investors and the capital allocation used to cover these losses.

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